

**Part A: Multiple Choice: 10 marks**

Identify the letter of the choice that best completes the statement.

- 1) When  $f(x) = 2x^3 - 11x^2 - 4x + a$  is divided by  $x + 2$ , the remainder is  $-4$ . What is the value of  $a$ ?  
 a) 48                      b) 56                      c)  $-48$                       d)  $-56$

- 2) Which of the following intervals represent the solution to the inequality  $(x - 3)(x + 1)(x + 3) < 0$ ?  
 a)  $x < 3$ , and  $-1 < x < 3$                       b)  $x \in (-3, -1) \cup 3, \infty)$   
 c)  $x \in (-\infty, -3) \cup (-1, 3)$                       d)  $-1 < x < 3$  and  $x > 2$

- 3) Which statement is false?  
 a) The real roots of the equation  $x^4 - 6x^2 - 27 = 0$  are the x-intercepts of the graph of the polynomial  $f(x) = x^4 - 6x^2 - 27$ .  
 b) The number of zeroes the graph of  $f(x) = x^3 + 3x^2 - x - 3$  has is 3.  
 c) The graph of a linear relation has a maximum of one x-intercept, and the graph of a quadratic function has a maximum of two x-intercepts.  
 d) If the graph of a particular polynomial function has exactly two x-intercepts, then the function must be quadratic.

- 4)  $y = x^3$  is stretched horizontally by a factor of 2, and then translated horizontally 3 units to the right. What is the equation of the resulting graph?  
 a)  $y = (2(x + 3))^3$                       b)  $y = \left(\frac{1}{2}(x - 3)\right)^3$   
 c)  $y = \left(\frac{1}{2}x\right)^3 - 3$                       d)  $y = (2x - 3)^3$

- 5) The real root(s) of the function  $f(x) = \frac{x^2 - 1}{x - 1}$  are:  
 a) 1 and  $-1$                       b) *only* 1                      c) *only*  $-1$                       d) None of these

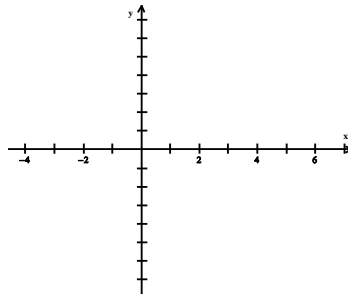
- 6) The vertical asymptotes for  $m(x) = \frac{-4(x - 3)}{3x^2 - 14x - 24}$  are:  
 a)  $x = 3, -\frac{4}{3}, 6$                       b)  $x = -\frac{4}{3}, -6$                       c)  $x = -6, 3$                       d)  $x = -\frac{4}{3}, 6$

- 7) The rational function  $q(x) = \frac{x^2 - 6x + 11}{x - 2}$  has an oblique asymptote at:  
 a)  $q(x) = x - 4$                       b)  $q(x) = x + 4$   
 c)  $q(x) = x - 8$                       d)  $q(x) = x + 8$

**Part B - Short Answer: 28 marks**

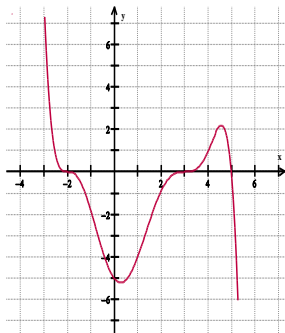
- 1) Consider the function  $h(x) = 4x^4(2x - 1)^2(2 - x)$ :  
 a) Determine the degree. \_\_\_\_\_  
 b) Determine its end behaviour. \_\_\_\_\_  
 c) Determine the zeroes. \_\_\_\_\_

- 2) Sketch a quartic function with:
- Zeroes -2 (order 2), 3 (order 2)
  - $x \rightarrow \pm\infty, y \rightarrow \infty$



- 3) Given the graph of  $f$  below:

- a) Determine an equation in factored form passing through the point  $(-3, 8)$ .

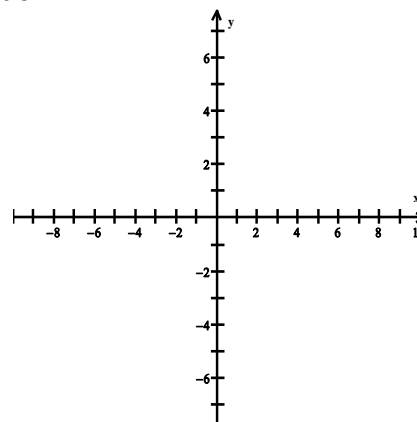


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- 4) Consider the following characteristics of a reciprocal quadratic function:

- $y \in (0, -\infty)$
- Vertical asymptotes  $x = 3$
- Slope is negative  $x < 3$   
and slope is positive  $x > 3$
- $f(0) = -2$

- a) Sketch the function.  
b) Provide a possible equation for  $f$ .



- 5) Find the diameter of a circle with an arc length of 32 cm and a subtended angle of  $72^\circ$ .

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- 6) Express as a single sine or cosine function.

$$\cos^2\left(\frac{3\theta}{2}\right) - \sin^2\left(\frac{3\theta}{2}\right)$$

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- 7) Write as a single trigonometric function then evaluate exactly:

$$\cos\frac{\pi}{7}\cos\frac{4\pi}{21} - \sin\frac{\pi}{7}\sin\frac{4\pi}{21}$$

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- 8) Express  $\sin\frac{38\pi}{6}$  in terms of a cosine function of an acute angle. Do not evaluate.

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- 9) If  $\tan\theta = \frac{-1}{\sqrt{3}}, -\pi \leq \theta \leq -\frac{\pi}{2}$   
find the exact value of  $\theta$ .

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- 10) State the domain for the function:

$$y = \log_2(10 - 3x)$$

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- 11) Solve for  $x$  ( to 3 decimal places )

a)  $x = \log_6 92^4$

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b)  $\log_x 44 = 1.572$

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- 12) Write as a single logarithm:  $3\log_2 m + \log_2 mn - \frac{1}{2}\log_2 n$

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**Part C - Full Solution: 50 marks**

1) Factor completely  $P(x) = 4x^3 + 12x^2 + 5x - 6$ .

2) Solve.

a)  $\frac{x^2 - x - 8}{x - 1} \geq 3$  using an interval chart.

b)  $\log_2(x - 7) + 3 = \log_2(2x)$

c)  $4^{2-3x} = 5^{8-x}$

3) Prove  $\frac{1 + \sec x}{\sec x} = \frac{\sin^2 x}{1 - \cos x}$

4) Given the function  $y = -3\sin[\pi x - 4] + 2$ , state the following characteristics:

a) Period \_\_\_\_\_

b) Range \_\_\_\_\_

c) phase shift \_\_\_\_\_

d) Axis of curve \_\_\_\_\_

5) Analyse the function  $g(x) = \frac{(4x-1)(x+3)(2x-1)}{x(x+3)(x+2)}$  using the characteristics listed below.

x-intercept(s): y-intercept(s): hole(s):	Equation of Asymptotes: Vertical: Horizontal: Slant:
End Behaviour:	Behaviour near asymptotes:

**Part D – Application – 23 marks**

6) Polluted water flows into a pond. The concentration of pollutant  $c$ , in the pond at time  $t$  minutes is modelled by the equation  $c(t) = 9 - \frac{90\,000}{10\,000 + 3t}$ , where  $c$  is measured in kilograms per cubic metre.

- a) When will the concentration of pollutant in the pond reach  $6\text{ kg/m}^3$ ?
- b) What will happen to the concentration of pollutant over time? Justify your reasoning.

7) An object is suspended from a spring which oscillates up and down. The distance from the highest point to the lowest point is 25 m. The object takes 5 seconds to complete 4 cycles. The object is initially stretched to its lowest point and then released.

- a) Write an equation that describes the distance ( $d$ ), in metres from the mean position with respect to the time,  $t$ , in seconds.
- b) When will this object be 3.5 m above the mean position for the first time?

8) The volume of air in the lungs during normal breathing is a sinusoidal function of time given by:

$$V = 300 \sin \frac{\pi}{2} t + 2500, \text{ where } t \text{ is the time in seconds and } V \text{ is in mL.}$$

- a) Find the average rate of change of the volume during the first 1.5 seconds.
- b) Estimate the instantaneous rate of change of volume at  $t = 2$  seconds. What does this tell you about the breathing at this time?
- c) Describe how the original graph would change if :
  - (i) the person breathes more rapidly
  - (ii) the person takes bigger breaths.

9) A thermocouple which is used to measure very high temperatures is placed on the element of a stove. The temperature of the element, in  $T$  degrees Celsius, can be modelled by the equation:

$$T = 150 \log 4t, \text{ where } t \text{ is in the time in seconds after the element is turned on}$$

- a) What is the temperature after 30 seconds?
- b) How long will it take for the temperature to reach  $375^\circ\text{C}$ ?