MHF4U: Review Package # 3 (Questions from old tests mixed together).

Part A: Short Answers

1) Sketch the function $y = -(x-2)(x+3)^2$.

2) Determine the horizontal asymptote for $f(x) = \dfrac{-5x^4}{x^3 - 9x}$

3) Factor completely. $5x^3 + 320y^6$.

4) Determine the remainder when $f(x) = 3x^3 - 5x^2 + x + 12$ is divided by $(x+2)$ without using long or synthetic division.

5) Evaluate $\log_{\sqrt{3}} 243$ without the use of a calculator.

6) Express as a single logarithm: $\log_7 x - 2\log_7 z - \frac{3}{4}\log_7 y$

7) Determine the coordinates of the hole for $g(x) = \frac{x^3-8}{x^2-4}$.

8) Convert $126°$ to exact radian measure.

9) Reduce $sin\frac{11\pi}{8}$ to the first quadrant using a co-related identity.

10) Simplify to a single trigonometric function $\frac{tan4x - tan5y}{1 - tan4xtan5y}$

11) State the domain of $f(x) = 2\log_3(x+4) - 1$.

12) State the range of $h(x) = -3(4^{x-3}) - 5$.

13) State the equation of the oblique asymptote of $y = \frac{4x^6 - 7x^3 - 2}{x^5 - 2x^2}$.

14) State the range of $y = 4\sec\left(2x - \frac{\pi}{5}\right)+1$.

15) Find the equation of a cosecant function that has a local maximum at $y = 5$, a local minimum at $y = -1$ and vertical asymptotes given by $\theta = \frac{\pi}{4} + \frac{\pi}{7}n, n \in I$.

16) Write an equation for a function that is the reciprocal of a quadratic and that has the following properties:

- Horizontal asymptote $y = 0$
- Vertical asymptote $x = 6$ and $x = -2$
- $y > 0$ on the intervals $(-\infty, -2)$ and $(6, \infty)$
- $f(0) = -\frac{1}{4}$

Part B: Full Solutions

1) Solve for x, $x \epsilon R$ and graph the solution set on a real number line.

a) $3x^3 + 4x^2 - 5x - 2 < 0$

b) $\frac{5}{2x+3} \geq 4$

2) Solve for $x, x\epsilon R$

a) $\log_2(x^2 - 6x) - \log(1 - x) = 3$

b) $36^{3x-1} = 6^{2x+5}$

c) $4^x + 15(4^{-x}) = 8$

d) $6^{x+3} = 7^{2x-1}$

e) $3(6^{2-x}) = 9175$

f) $sin2x + \sqrt{2}sinx = 0,\ x \in [-\pi, 2\pi]$

3) Graph the function $f(x) = \frac{5-x}{x^2-16}$, by first determining the intercepts, equations of asymptotes and behavior of the function around all asymptotes.

4) Express as a single logarithm, then evaluate $\log_4 \sqrt{40} + \log_4 \sqrt{48} - \log_4 \sqrt{15}$

5) Graph $y = \log_2(4 - x)^{-2}$.

6) Determine the exact average rate of change of the function $y = 2sin\left(x - \frac{\pi}{6}\right) + 1$ on the interval $\frac{\pi}{2} \leq x \leq 3\pi$.

7) Using identities, evaluate exactly

a) $sin\frac{9\pi}{8}$

b) $cos\frac{11}{12}\pi$

8) Prove $secx = \frac{2[cosxsin2x - sinxcos2x]}{sin2x}$

9) The sound intensity of a soft whisper is about $\frac{1}{200{,}000}$ of the sound intensity of a shout. What is the decibel level of a whisper if a shout has a loudness level of about 85 dB?

10) Jackson wants to invest his considerable chess prize money in a premium savings account earning 2.25% compounded semi-annually. How long will it take his initial deposit (P) to quadruple in value? (Express answer correct to 1 decimal place)

11) The tides at Cape Capstan, N.B, change depth of the water in the harbour. On one day in October, the tides have a high point of approximately 10 m at 2 p.m. and a low point of approximately 1.2 m at 8:15 p.m. A particular sailboat has a draft of 2 m. This means it can only move in water that is at least 2 m deep. The captain of the sailboat plans to exit the harbour at 6 p.m. Create a sinusoidal function to model the problem, and use it to determine whether the sailboat can exit the harbour safely at 6 p.m. Assume $t = 0$ is midnight.