

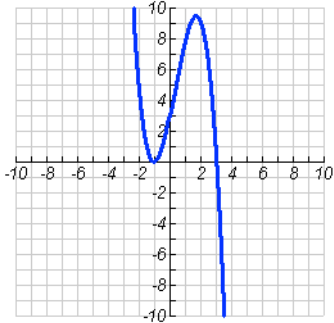
MHF 4U EXAM PREP QUESTIONS - Package #2

POLYNOMIAL FUNCTIONS (Unit 1 and 2)

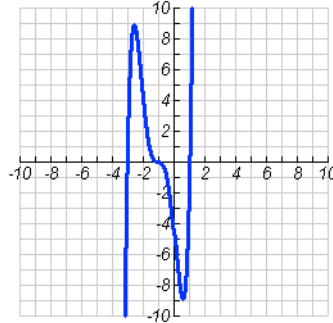
- Divide and state restrictions on the variables.
 - $(x^2 + 10x + 16) \div (x + 2)$
 - $(y^3 + y^2 - 2) \div (y - 1)$
 - $(3y^3 - 2y^2 + 12y - 9) \div (y^2 + 2)$
 - $(15x - 4x^3 - 9x^2 + 3x^4 - 4) \div (3x - 4)$
- Determine the remainder without dividing.
 - $(x^2 + 5x - 8) \div (x - 2)$
 - $(3m^2 + 7m + 1) \div (m + 3)$
 - $(2x^2 + 5x + 11) \div (2x - 1)$
 - $(8y^3 + 12y^2 - 4y + 5) \div (2y + 3)$
- Find the value of k if the remainder is -1 when $(x^3 - 3x^2 - 6x + k)$ is divided by $(x + 2)$.
- Grace has recently written a book. Her publisher models the cumulative total number of copies sold, $C(t)$, t days after the book is released by the formula, $C(t) = 0.5t^2 + 20t + 50; t \in [0, 30]$. Divide $C(t)$ by $t - 10$ and state the quotient and the remainder. What does the quotient represent in this case?
- Find the integer k such that $(x - 2)$ divides $(3x^3 - 2x^2 + x + k)$ evenly.
- Factor completely.
 - $(x^3 - x^2 - 5x - 3)$
 - $(x^3 + 5x^2 + 3x - 4)$
 - $(2x^3 - x^2 - 2x + 1)$
 - $(3x^3 + 13x^2 - 16)$
- Solve and check.
 - $y^3 = 9y$
 - $n^3 - 3n - 2 = 0$
 - $3w^2 + 11w = 2w^3 + 6$
 - $3x^2 - 2 = 8x^3 - 7x$
- A cereal box is a rectangular prism with a volume of 2500cm^3 . The box is 4 times as wide as it is deep, and 5cm taller than it is wide. What are the dimensions of the box?
- Graph each function and determine all key properties (domain, range, intercepts, intervals where $f(x) > 0, f(x) \leq 0$, end behaviour).
 - $f(x) = x(x + 4)(x - 4)$
 - $f(x) = -(x + 2)(x - 3)(x - 5)$
 - $f(x) = -x(x - 3)(x + 1)(x + 4)$
 - $f(x) = x^2(x^2 - 9)$
 - $f(x) = x^4 - 2$
 - $f(x) = x^3 - x^2 - 2x - 3$
- Solve each inequality.
 - $x(x + 3) > 0$
 - $(x + 4)(x - 1) < 0$
 - $(x + 2)(x - 2)(x + 4) \geq 0$
 - $x^3 - 2x^2 - 5x > 6$
- A quartic function has x -intercepts of $-5, -1, 2,$ and 4 . The point $(3, -8)$ lies on the curve. Determine the equation of this function.

12. Determine the equation of each function graphed below.

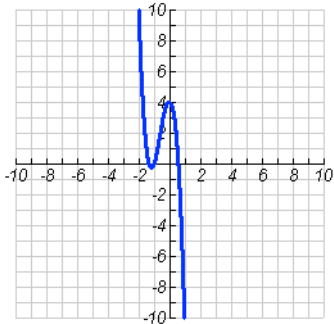
a)



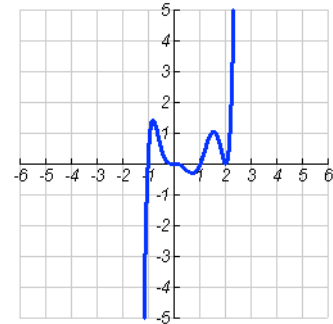
b)



c)



d)



ANSWERS

1a) $x + 8, x \neq -2$ b) $y^2 + 2y + 2, y \neq 1$ c) $3y - 2, R: 6y - 5$ d) $x^3 - 3x + 1, x \neq 4/3$

2a) 6 b) 7 c) 14 d) 11 3) 7 4) $Q(t) = 0.5t + 25, R(t) = 300$ 5) $k = -18$

6a) $(x + 1)^2(x - 3)$ b) $(x + 4)(x^2 + x - 1)$ c) $(x - 1)(x + 1)(2x - 1)$ d) $(x - 1)(x + 4)(3x + 4)$

7a) $0, \pm 3$ b) $-1, 2$ c) $\frac{1}{2}, 3, -2$ d) $1, \frac{-5 \pm \sqrt{65}}{10}$ 8) 25cm x 20cm x 5cm

10a) $x \in (-\infty, -3) \cup (0, \infty)$ b) $x \in (-4, 1)$ c) $x \in [-4, -2] \cup [2, \infty)$ d) $x \in (3.76, \infty)$

11) $y = \frac{1}{4}(x + 5)(x + 1)(x - 2)(x - 4)$ 12a) $y = -(x + 1)^2(x - 3)$ b) $y = -2(x + 1)^3(x + 3)(x - 1)$

c) $y = a(2x + 3)(x + 1)(2x - 1)$ d) $y = ax^3(x - 2)^2(x + 1)(x - 1)$

RATIONAL FUNCTIONS (Unit 3)

1. Determine the equation(s) of all vertical asymptotes and sketch the curve near the asymptotes.

a) $f(x) = \frac{4}{x - 2}$

b) $f(x) = \frac{5}{(x - 3)^2}$

c) $f(x) = \frac{x}{1 - x^2}$

d) $f(x) = \frac{4x^4 + 8x}{x^2 + 6x + 8}$

e) $f(x) = \frac{x + 3}{x^4 - 9x^2}$

f) $f(x) = \frac{3}{x^3 - 4x}$

2. Determine the equation(s) of all vertical, horizontal or slant asymptotes. Sketch each function by first stating important characteristics. (Intercepts, end behaviour, discontinuities (hole), etc...)

$$\begin{array}{lll} \text{a) } f(x) = \frac{2x-3}{5-x} & \text{b) } f(x) = \frac{x^2+4}{x} & \text{c) } f(x) = \frac{9x-6}{1-3x} \\ \text{d) } f(x) = \frac{x^2+5x+4}{x-1} & \text{e) } f(x) = \frac{x^3+5x^2+3x+10}{x^2+2} & \text{f) } f(x) = \frac{6x^2+4x+1}{5-3x^2} \end{array}$$

3. A piece of machinery depreciates in value, V , in dollars, over time, t , in months. The value is given by the function, $V(t) = 5000 - \frac{2000t^2}{(t+2)^2}$.
- Determine the values of the machinery after i) 6 months, and ii) 10 years.
 - Describe the end behaviour of this function (what happens to the value of the machinery in the long run)?
 - Will the machinery ever have a value of \$0? Explain.

ANSWERS

- 1a) $x = 2$ b) $x = 3$ c) $x = \pm 1$ d) $x = -2, x = -4$ e) $x = 0, x = \pm 3$ f) $x = 0, x = \pm 2$
 2a) $x = 5, y = -2$ b) $x = 0, y = x$ c) $x = \frac{1}{3}, y = -3$ d) $x = 1, y = x + 6$ e)
 f) $x = \pm \sqrt{\frac{5}{3}}, y = -2$
 3a) i) 3875, ii) 3065.04 b) approaches a value of \$3000 c) no, VA at $y=3000$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS (UNIT 5 & 6)

1. Sketch the following
- $$\begin{array}{lll} \text{(a) } y = 3(2)^x & \text{(b) } y = -4(2)^x & \text{(c) } y = -(2)^{3x} \quad \text{(d) } y = 2^{2x+6} \\ \text{(e) } y = 2^{-x} + 4 & \text{(f) } y = 2^{2x-4} - 3 & \text{(g) } y = 4\left(\frac{1}{2}\right)^x \end{array}$$
2. Sketch the following
- $$\begin{array}{llll} \text{(a) } 2 \log x & \text{(b) } y = -\log x + 2 & \text{(c) } y = \log(3x) & \text{(d) } y = \log(x+4) \\ \text{(e) } y = \log(2x-6) & \text{(f) } y = \log\left(\frac{1}{3}x+2\right) & \text{(g) } y = -\log(-2x)+2 & \end{array}$$
3. Express in logarithmic form
- $$\text{(a) } 5^2 = 25 \quad \text{(b) } 6^0 = 1 \quad \text{(c) } 49^{\frac{1}{2}} = 7 \quad \text{(d) } 125^{\frac{2}{3}} = 25$$
4. Express in exponential form
- $$\text{(a) } \log_3 1 = 0 \quad \text{(b) } \log_6\left(\frac{1}{36}\right) = -2 \quad \text{(c) } \log_9 27 = \frac{3}{2}$$
5. Evaluate the following
- $$\text{(a) } \log_7 49 \quad \text{(b) } \log_2 16 \quad \text{(c) } \log_3 \sqrt{27} \quad \text{(d) } \log_2 \sqrt[3]{32} \quad \text{(e) } \log_{17} 1 \quad \text{(f) } \log_3 \frac{1}{81}$$

(g) $6^{-2\log_6 8}$ (h) $7^{2\log_7 6}$ (i) $\log_3(9\sqrt{27})$ (j) $\log_4(64 \times \sqrt[3]{16})$ (k) $\log_5\left(\frac{\sqrt[3]{25}}{625}\right)$

(l) $\log_{12} 576 - \log_{12} 4$ (m) $\log_6 18 + \log_6 12$

6. Solve

(a) $\log x = \log 5 + 2\log 3$ (b) $\log \sqrt{x} = \log 1 - 3\log 2$ (c) $\log_3 x - \log_3 4 = \log_3 12$

(d) $\log_6(x+3) + \log_6(x-2) = 1$ (e) $\log_7 x = 3\log_7 4$

(f) $\log_5(7x+1) - \log_5(x-1) = 2$

7. Express as a single log:

(a) $\frac{1}{2}\log_5 x + \frac{1}{3}\log_5 y - \frac{1}{4}\log_5 z$ (b) $\frac{1}{2}[\log_4 x + 3\log_4 y] - 2[\log_4 a + \log_4 b]$

8. Solve

(a) $3^{2x-1} = 5$ (b) $2^{2x} + 3(2^x) - 10 = 0$ (c) $6^{2x} - 2(6^x) - 15 = 0$

(d) $\left(\frac{1}{64}\right)^{x+2} = 8^{2x}$

(e) $6(10^{2x}) + 10^x - 2 = 0$

9. A sample of 500 cells in a medical research lab doubles every 20 min.

a) Determine a formula for the number of cells at time t , where t is measured in minutes.

b) How long will it take for the population to reach 18 000? Answer correct to 2 decimal places.

10. In 1987 there were about 130 000 cell phone users in Canada. In 1999, there were about 10 million cell phone users. What is the percent increase per year? Answer correct to 2 decimal places.

11. A sample of 700 cells in a medical research lab triples every 30 min.

a) Determine a formula for the number of cells at time t .

b) How long will it take for the population to reach 18 000? Answer correct to 2 decimal places

12. A sample of radioactive iodine-131 atoms has a half-life of about 8 days. Suppose that 1 000 000 iodine-131 atoms are initially present.

a) Determine a formula for the number of atoms at time t , where t represents number of days

b) How long will it take for the sample to reach 180 000 atoms? Answer correct to 2 decimal places.

13. A new car costs \$23 000. In 5 years it will be worth \$9500. What is the rate of depreciation per year? Answer in percent, correct to 2 decimal places.

14. Most of Canada's earthquakes occur along the west coast. In 1949, there was an earthquake in the Queen Charlotte Islands that had a magnitude of 8.1 on the Richter Scale. In 1997 there was an earthquake in south-western B.C. with a magnitude of 4.6 on the Richter Scale. How many times as intense as the 1997 earthquake was the 1949 earthquake? Answer correct to 2 decimal places.

15. The loudness L in decibels ($\frac{1}{10}$ of a bel) of a sound of intensity I is defined to be

$$L = 10 \log \frac{I}{I_0} \quad \text{where } I_0 \text{ is the minimum intensity audible to the human ear}$$

- The loudness level of a heavy snore is 69 dB. How many times is this more intense than conversational speech at 60 dB? Answer correct to 2 decimal places.
- Sound is 316 times less intense if earplugs are worn. What would the decibel level of snoring be if earplugs were worn? Answer correct to the nearest dB.

16. Given the function $y = 4^x$, determine:

- the average rate of change from $t=5$ seconds to $t=6$ seconds.
- the instantaneous rate of change at $t=5$ seconds.

ANSWERS

3. (a) $\log_5 25 = 2$ (b) $\log_6 1 = 0$ (c) $\log_{49} 7 = \frac{1}{2}$ (d) $\log_{125} 25 = \frac{2}{3}$ 4(a) $3^0 = 1$ (b) $6^{-2} = \frac{1}{36}$

(c) $9^{\frac{2}{3}} = 27$ 5.(a) 2 (b) 4 (c) $\frac{3}{2}$ (d) $\frac{5}{3}$ (e) 0 (f) -4 (g) $\frac{1}{64}$ (h) 36 (i) $\frac{7}{2}$ (j) $\frac{11}{3}$ (k) $\frac{-10}{3}$ (l) 2

(m) 3 6.(a) $x = 45$ (b) $x = \frac{1}{64}$ (c) $x = 48$ (d) $x = 3$ (e) 64 (f) $\frac{13}{9}$ 7. (a) $\log\left(\frac{\sqrt{x}\sqrt[3]{y}}{\sqrt[4]{z}}\right)$ (b)

$\log_4\left(\frac{\sqrt{xy^3}}{a^2b^2}\right)$ 8. (a) $x = 1.23$ (b) $x = 1$ (c) $x = 0.9$ (d) $x = -1$ (e) $x = -0.3$ 9(a)

$N(t) = 500(2)^{\frac{t}{20}}$ (b) 103.40 min 10.. 43.61% 11. (a) $N(t) = 700(3)^{\frac{t}{30}}$ (b) 88.67 min

12. (a) $N(t) = 1000000\left(\frac{1}{2}\right)^{\frac{t}{8}}$ (b) 19.79 days 13.16.21% 14. 3162.28 15. (a) 7.94 (b) 44 dB

16. (a) 3072 (b) 1419.57