

## Expressions

1. Factor fully.

a)  $x^3 + 3x^2 - 25x - 75$

b)  $x^3 + x^2 - 14x - 24$

c)  $64x^3 + 27y^3$

d)  $30x^3 + 17x^2 - 8x - 4$

e)  $x^6 - 124x^3 - 125$

f)  $8x^6 - 125y^6$

2. Find the exact value of the following.

a)  $7^{\log_7 \sqrt{5}}$

b)  $\log_{64} \sqrt[6]{8}$

c)  $\log_8 6 - \log_8 3 + \log_8 4$

d)  $\log_9 (3^7 \times \sqrt[5]{81})$

3. Write as a single logarithm.

a)  $a \log_5 (x - 7) - \frac{2}{3} \log_5 w + 2$

b)  $(x + 1) - \log(x - 7)$

c)  $\frac{1}{2} \log_6 12 + \log_6 8 - \log_6 2$

d)  $\log_2 10 - \frac{1}{3} \log_2 9$

4. Convert to degrees.

a)  $\frac{11\pi}{15}$  radians

b) 56 radians

5. Convert to radians.

a)  $420^\circ$

b)  $-24^\circ$

6. Find the exact value of the following.

a)  $\cos \frac{3\pi}{4}$

c)  $\csc \left( \frac{-3\pi}{2} \right)$

b)  $\tan \frac{11\pi}{6}$

d)  $\sin \frac{7\pi}{12}$

7. Given:  $\sin A = \frac{-6}{7}$ ,  $\frac{3\pi}{2} \leq A \leq 2\pi$ , and  $\tan B = \frac{2}{3}$ ,  $\pi \leq B \leq \frac{3\pi}{2}$

Find the exact value of the following.

a)  $\sec A$

c)  $\sin(A + B)$

b)  $\cos 2B$

d)  $\tan(A - B)$

8. Given:  $f(x) = \frac{1}{x-5}$  and  $g(x) = x^2 + 8$

Find:

a)  $(f - g)(x)$

e)  $(g \circ g)(x)$

i)  $(g - f)(3)$

b)  $\left( \frac{g}{f} \right)(x)$

f)  $f^{-1}(x)$

j)  $(fg)(-1)$

c)  $(f \circ g)(x)$

g)  $g^{-1}(x)$

k)  $(f \circ g)(5)$

d)  $(g \circ f)(x)$

h)  $(f \circ f^{-1})(x)$

l)  $(g \circ f)(5)$

## Answers

1.a.

b.

c.  $(4x+3y)(16x^2-12xy+9y^2)$

d.  $(5x+2)(2x-1)(3x+2)$

e.  $(x+1)(x-5)(x^2-x+1)(x^2+5x+25)$

$(\sqrt{2}x - \sqrt{5}y)(\sqrt{2}x + \sqrt{5}y)$

f.  $(4x^4 + 10x^2y^2 + 25y^4)$

2a.      b.      c.      d.

3a.

b.

c.  $\log_6 8\sqrt{3}$

d.  $\log_2 \frac{10}{\sqrt[3]{9}}$

4a.  $132^\circ$  b.

5a.      b.

6a.      b.      c. Und d.

7a.      b.      c.

d.

8a.

b.

c.

d.

e.

f.

g.

h. x i.

j.

k.

l. und

## Equations and Identities

9. Solve. Exact answers are required, where possible. Otherwise, express answers correct to one decimal place.

- a)  $x^3 - 3x^2 = 4x - 12$       h)  $\log_9(x - 5) + \log_9(x + 3) = 1$   
 b)  $x^3 - 5x = 5x^2 - 1$       i)  $3^{2x} - 2(3^x) - 15 = 0$   
 c)  $x^3 + 4x^2 + 9x + 10 = 0$       j)  $2 \cos 2x = 1 \quad (0 \leq x \leq 2\pi)$   
 d)  $x + \frac{1}{x-4} = 0$       k)  $\sin^2 x - 2 \sin x - 3 = 0 \quad (0 \leq x \leq 2\pi)$   
 e)  $\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2 - 4x + 3}$       l)  $\cos 2x = \cos x \quad (0 \leq x \leq 2\pi)$   
 f)  $4(7^{x-2}) = 8$       m)  $\sqrt{2} \tan x \cos x = \tan x \quad (0 \leq x \leq 2\pi)$   
 g)  $\log_4(x + 3) = 2$       n)  $(4^2)(2^{2x-3}) = (16^{x-2})\left(\frac{1}{\sqrt{2}}\right)$   
 o)  $\log_4(x + 3) = 2$       p)  $\log_7(x + 2) = 1 - \log_7(x - 4)$

10. Solve.

- a)  $x(x+1)(x-2)(x-4) > 0$       d)  $\frac{x+2}{x^2-9} \geq 0$   
 b)  $(x+7)^2(x-3)^3 < 0$       e)  $\frac{5}{x+3} + \frac{3}{x-1} < 0$   
 c)  $2x^3 + 3x^2 - 11x \geq 6$

11. Prove.

- a)  $\cos \theta + \sin \theta = \frac{1 + \tan \theta}{\sec \theta}$       c)  $\frac{\csc^2 \theta - 2}{\csc^2 \theta} = \cos 2\theta$   
 b)  $\frac{1}{1 - \sec \theta} + \frac{1}{1 + \sec \theta} = -2 \cot^2 \theta$       d)  $\cos^2 2\theta - \cos^2 \theta = \sin^2 \theta - \sin^2 2\theta$   
 e)  $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) = -\cot x$   
 f)  $\cos(x + y) \cos(x - y) = \cos^2 x + \cos^2 y - 1$

12. If  $\log_b a = \frac{1}{x}$  and  $\log_a \sqrt{b} = 3x^2$ , show that  $x = \frac{1}{6}$

13. If  $h^2 + k^2 = 23hk$  where  $h > 0, k > 0$ , show that  $\log\left(\frac{h+k}{5}\right) = \frac{1}{2}(\log h + \log k)$

## Graphs

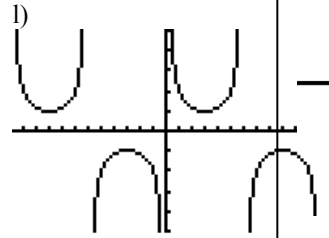
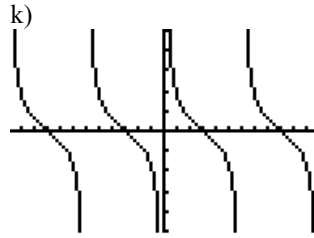
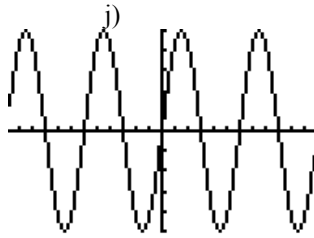
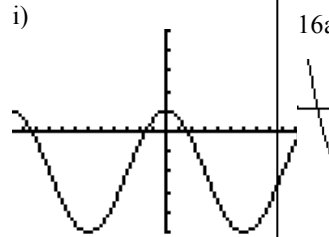
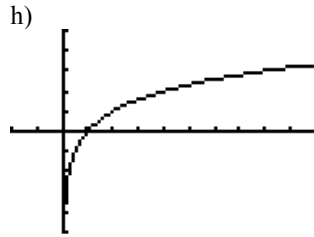
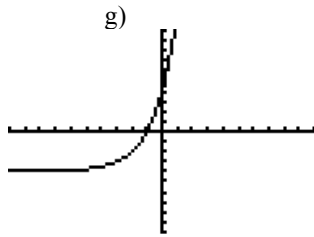
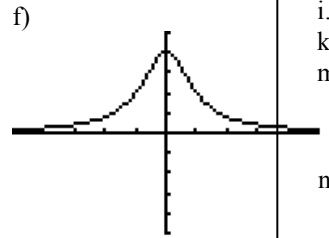
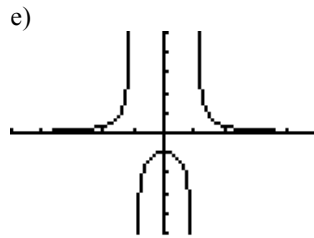
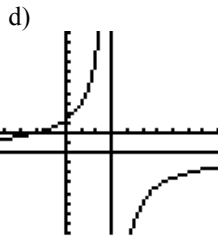
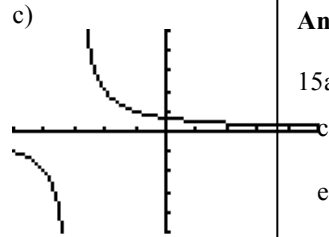
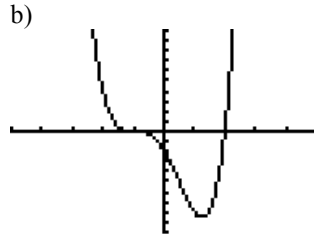
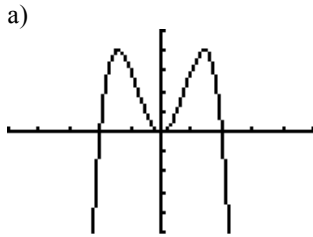
14. Determine whether each of the following functions is even, odd, or neither.

- a)  $f(x) = \frac{1}{x^3+1}$       b)  $g(h) = 2h^4 + 3h^2$       c)  $p(x) = 2x(x-4)^2(x+4)^2$

## Answers

- 9a. 3, 2, -2    b. -1,  
 c. -2, -1±2i    d.      e.  
 f.      g. 13    h. 6    i.  
 j.      k.  
 l.  
 m.  
 10a.  $x < -1, 0 < x < 2, x > 4$   
 b.  $x < -7, -7 < x < 3$   
 c.  $-3 \leq x \leq -0.5, x \geq 2$   
 d.  $-3 < x \leq -2, x > 3$   
 e.  $x < -3, -0.5 < x < 1$

15. Write an equation to represent each function.



m) Consider the above graphs. Which functions:

- i) are even?
- ii) are odd?
- iii) have inverses that are functions?

16. Graph the following functions. Label all key features.

a)  $y = -x(x - 3)(x + 4)$

e)  $y = 3^{x+2} - 1$

b)  $y = (x - 2)^2(x + 3)^3$

f)  $y = \log_2(8x^2)$

c)  $y = \frac{-2}{x - 1}$

g)  $y = 2 \sin\left(x - \frac{\pi}{3}\right), (-2\pi \leq x \leq 2\pi)$

d)  $y = \frac{5x - 3}{2x + 1}$

h)  $y = \cos\left(\frac{1}{2}x + \frac{\pi}{4}\right) - 1, (-2\pi \leq x \leq 2\pi)$

17. When is the function  $f(x) = \frac{4}{x-1} - 3 + \frac{-3x^2}{5-4x-x^2}$  below the horizontal asymptote?

18. State the characteristics (properties) of each function:

a)  $f(x) = -2 - 3\cos\left(3x - \frac{\pi}{4}\right)$

b)  $y = \cot\left(x - \frac{\pi}{6}\right)$

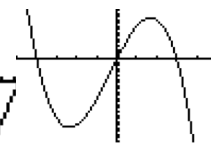
**Answers**

15a.  $y = -x^2(x-2)(x+2)$     b.  $y = (x-2)(x+1)^3$

- c.
- d.
- e.
- f.
- g.
- h.
- i.  $y = 3\cos x - 2$
- j.  $y = 5\sin 2x$
- k.  $y = \cot x$
- l.  $y = \csc x$
- m. even: a, e, f, l
- odd: j, k, l
- inverse fun: c, d, g, h
- n. c:
- d:
- g:
- h:  $y = 2^x$

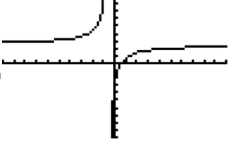
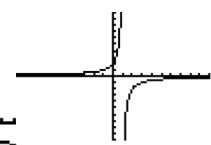
16a.

b.



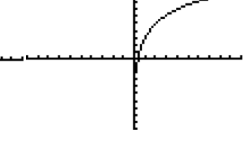
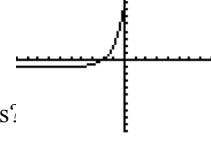
c.

d.



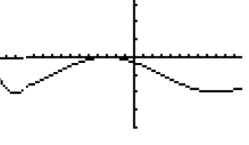
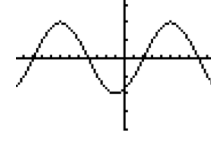
e.

f.



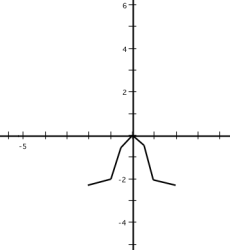
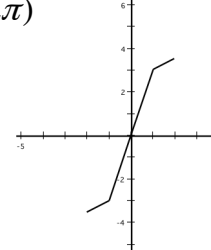
g.

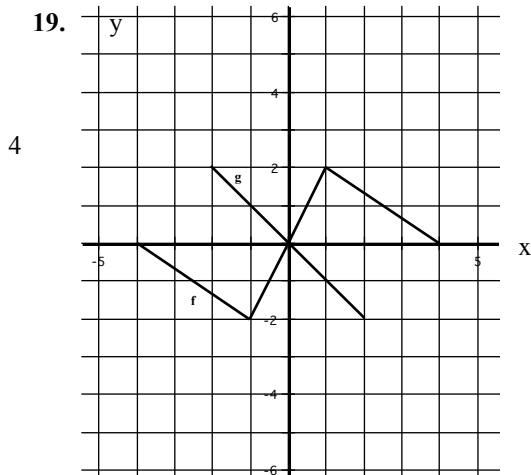
h.



19a.

b.





Given the graphs of  $y=f(x)$  and  $y=g(x)$ , graph:

- a)  $(f - g)(x)$
- b)  $(fg)(x)$
- c)  $\left(\frac{f}{g}\right)(x)$
- d)  $f^{-1}(x)$
- e)  $g^{-1}(x)$

Which functions are even? Odd?

**Applications**

- 20. When  $x^4 - 4x^3 + ax^2 + bx + 1$  is divided by  $(x-1)$ , the remainder is 7. When it is divided by  $(x+1)$ , the remainder is 3. Determine the values of  $a$  and  $b$ .
- 21. An open box, no more than 5 cm in height, is to be formed by cutting four identical squares from the corners of a sheet of metal 25 cm by 32 cm, and folding up the metal to form sides. The capacity of the box must be  $1575 \text{ cm}^3$ . What is the side length of the squares removed?
- 22. Consider all rectangles with an area of  $200 \text{ m}^2$ . Let  $x$  be the length of one side of the rectangle.
  - a) Express the perimeter as a function of  $x$ .
  - b) Find the dimensions of a rectangle whose perimeter is 70 m.
- 23. Determine the intercepts and the equations of all asymptotes of  $y = \frac{x^3 - 2x^2 - x + 2}{x^2 - x - 6}$  then sketch.
- 24. Pollution affects the clarity of water. The intensity of light below a particular polluted river's surface is reduced by 5% for each metre below the surface of the water.
  - a) Write an exponential function to model this relationship.
  - b) What percent of the original intensity of light penetrates to 4 m below the surface of the water?
  - c) At what depth does 40% of the original light intensity remain?
- 25. Energy is needed to transport a substance from outside a living cell to inside the cell. This energy is measured in kilocalories per gram molecule, and is given by:  $E = 1.4(\log C_1 - \log C_2)$ , where  $C_1$  represents the concentration of the substance outside the cell and  $C_2$  represents the concentration of the substance inside the cell.
  - a) Rewrite the formula as a single logarithm.
  - b) Find the energy needed to transport the exterior substance into the cell if the concentration of the substance outside the cell is double the concentration inside the cell.

c.

d.

e.

f. even: bc      odd: ae  
not a function: d

20.  $a=3, b=6$   
 21. 3.5 cm  
 22a.  
     b. 27.8 x 7.2 m  
 23. x-int=1, -1, 2  
     y-int=  
     VA:  $x=3, x=-2$   
     Obl A:  $y=x-1$   
 24a.  
     b. R  
     c. m  
 25a.  
     b. 0.42    c. neg – cell gains energy

- c) What is the sign of E if  $C_1 < C_2$ ? Explain what this means in terms of the cell.
26. Strontium-87 is used in the study of bones. After one hour, 78.1 mg remains of a 100 mg substance. What is the half-life of Strontium-87?
27. A circular arc has length 3 cm, and the radius of the circle is 2 cm. What is the measure of the angle subtended by the arc, in both radians and in degrees?
28. Tidal forces are greatest when the Earth, the sun and the moon are in line. When this occurs at the Annapolis Tidal Generating Station, the water has a maximum depth of 9.6 m at 4:30 pm and a minimum depth of 0.4 m 6.2 h later.
- Write an equation for the depth of the water at any time  $t$ .
  - Calculate the depth of water at 9:30 am and at 6:45 pm.
29. The function,  $S(t) = -3 \cos \frac{\pi}{12} t + 22$ , represents the surface temperature  $S$  of a pond, in degrees Celsius, and  $t$  represents the number of hours since sunrise at 06:00. At what time is the surface temperature of the water  $20^\circ\text{C}$ ?
30. The meat department manager discovers that he could sell  $m(x)$  kg of ground beef in a week, where  $m(x) = 14700 - 3040x$ , if he sold it at  $\$x/\text{kg}$ . He pays the supplier  $\$3.21/\text{kg}$  for the beef.
- Determine an algebraic expression for  $P(x)$ , where  $P$  represents the total profit from selling ground beef for one week.
  - Find  $P^{-1}(x)$ .
  - What selling price will earn  $\$1900$  in profit for the week?
31. Estimate IRC of each function at the given  $x$  value using a centered interval of 0.001.
- $f(x) = x^3 + x^2$  at  $x = 2$
  - $f(x) = -x^4 + 1$  at  $x = 3$
32. A Ferris wheel with radius 10 m makes 2 rotations in 4 minutes. What is the speed of the Ferris wheel in metres per second.

**Answers**

26. 2.8 h

27.

28a.

b. 1.2m, 7.7m

29. 3:12, 20:46

30a.

b.

c.  $\$4.21/\text{kg}$  or  $\$3.83/\text{kg}$