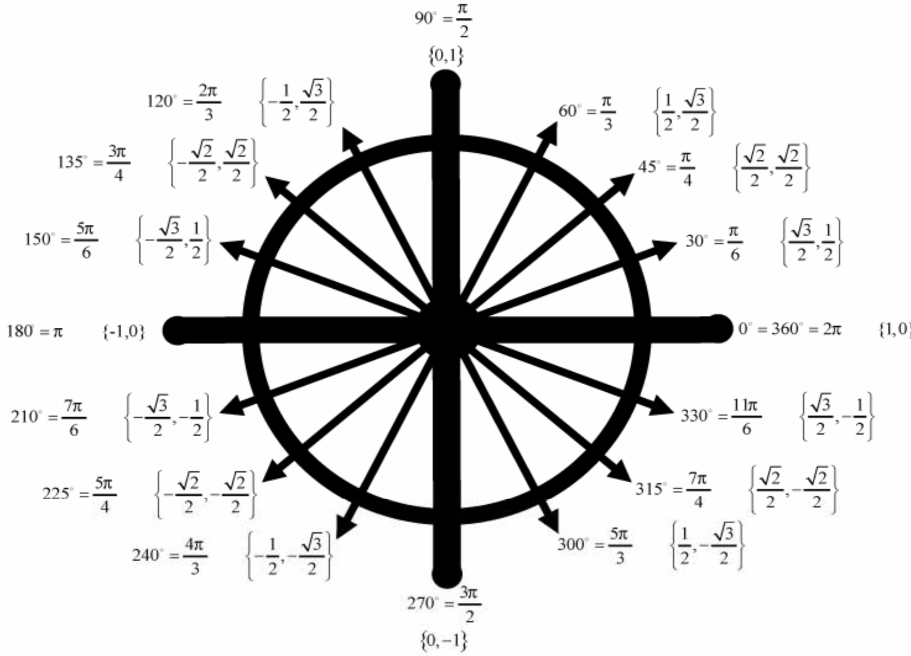
Pure Math 30:

TRIGONOMETRY I



LESSON EIGHT

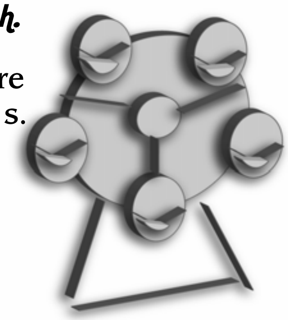
Applications of Trigonometry

Pure Math 30: EXPLAINED!

By Barry Mabillard

Trigonometry Lesson 8:
Part I – Ferris Wheels

One of the most common application questions for graphing trigonometric functions involves Ferris wheels, since the up and down motion of a rider follows the shape of a sine or cosine graph.



Example: A Ferris wheel has a diameter of 30 m, with the centre 18 m above the ground. It makes one complete rotation every 60 s.

a) Draw the graph of one complete cycle, assuming the rider starts at the lowest point.
b) Find the cosine equation of the graph.
c) What is the height of the rider at 52 seconds?
d) At what time(s) is the rider at 20 m?

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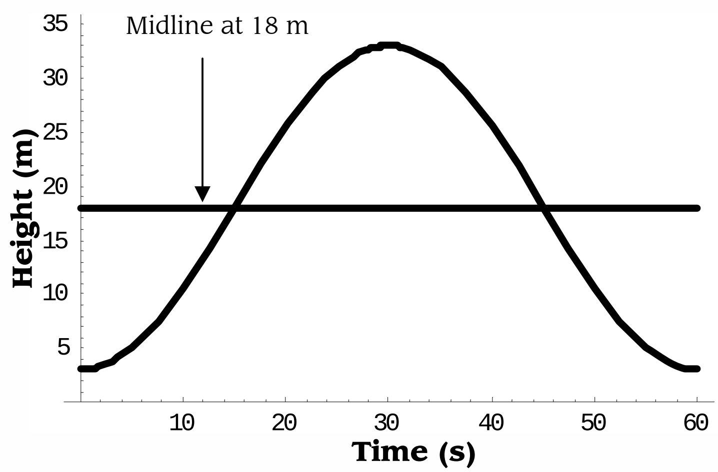
a. When you look at a Ferris wheel it makes a circular motion. Do <u>NOT</u> draw a circle as your graph. The graph represents the up & down motion over time.
In the first half of the graph, we can see the person will go up from 3 m to 33 m in 30 seconds. In the second half, the person will go back down to 3 m.



b. ***a-value:*** We know the diameter of the wheel is 30 m, so the radius (which is the same as amplitude) will be 15 m.

b-value: The period is 60 s, so:

$$b = \frac{2\pi}{P} = \frac{2\pi}{60} = \frac{\pi}{30}$$

c-value: There is none in the cosine pattern.

d-value: The centre of the wheel is 18 m above the ground, so that will be the midline

The cosine pattern is upside down, so you need to put a negative in front.

$$h(t) = -15\cos\frac{\pi}{30}t + 18$$

c. ***Method 1:*** Plug 52 s in for time in either the cosine equation. Simplify and calculate with your TI-83 in radian mode

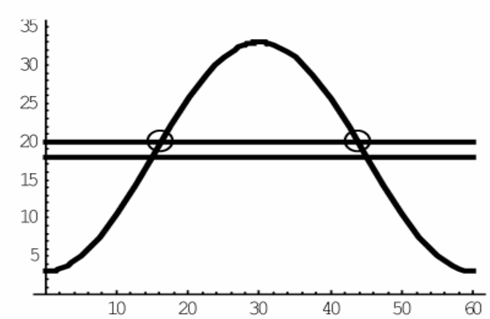
$$h(t) = -15\cos\frac{\pi}{30}t + 18$$

$$h(52) = -15\cos\frac{\pi}{30}52 + 18$$

$$h(52) = 7.96 \text{ m}$$

Method 2: Put your calculator in radian mode and graph either the cosine function.
2nd Trace → Value → x = 52
This will calculate the height directly from the graph.

d. To find the times the rider is at 20 m, graph a horizontal line in your TI-83 at y = 20, then find the points of intersection.



The rider has a height of 20 m at 16.3 s and 43.7 s

Trigonometry Lesson 8:

Part I – Ferris Wheels

In some Ferris wheel questions, you will be given the rotational speed in units of revolutions per second or revolutions per minute. You can use these values to find the period as follows:

Example: **A Ferris wheel rotates at 4 rev/min. What is the period in minutes & seconds?**

Think of it this way: there are 60 seconds in a minute. If it completes four revolutions, each one must take only 15 seconds. Thus, the period is 15 s.

Conversion Reminder:
Minutes → Seconds: Multiply by 60
Seconds → Minutes: Divide by 60

Ferris Wheel Questions

1. **A Ferris wheel has a radius of 35 m and starts 2 m above the ground. It rotates once every 53 seconds.**

a) Determine the cosine equation of the graph, if the rider gets on at the lowest point.

b) Sketch two complete cycles of a graph representing the height of a rider above the ground, assuming the rider gets on the Ferris wheel at the lowest point.

c) What is the height of the rider at 81 seconds?

d) At what time does the rider first reach a height of 51 m?

2. **A Ferris wheel makes 4 revolutions in one minute. The diameter of the Ferris wheel is 28 m, and a rider boards from a platform at a height of 30 m above the ground. The ride lasts 30 seconds.**

a) Determine the cosine equation. (*Hint*: Think about where the rider is starting in this question.)

b) Sketch the graph for the complete ride.

c) At what times is the rider at a height of 20 m?

3. **A water wheel on a paddleboat has a radius of 1 m. The wheel rotates once every 1.46 seconds and the bottom 0.3 m of the wheel is submerged in water. (Consider the water surface to be the x-axis)**

a) Determine the cosine equation of the graph, starting from a point at the top of the wheel.

b) Graph the height of a point on the wheel relative to the surface of the water, starting from the highest point.

c) How long is a point on the wheel underwater?

4. **The bottom of a windmill is 8 m above the ground, and the top is 22 m above the ground. The wheel rotates once every five seconds.**

a) Determine the cosine equation of the graph, starting from a point at the bottom of the windmill.

b) Draw the graph of two complete cycles.

c) What is the height of the point after 4 seconds?

d) For how long *(over the course of both cycles)* is the point above 17 m?

5. **A point on an industrial flywheel experiences a motion described by the formula:** $h(t) = 13\cos\frac{2\pi}{0.7}t + 15$

a) What is the maximum height of the point?

b) At what time is the maximum height reached?

c) What is the minimum height of the point?

d) At what time is the minimum height reached?

e) How long, within one cycle, is the point lower than 6 m above the ground?

f) What is the height of the point if the wheel turns for 2 hours and 20 minutes?

1. a)

a-value: The amplitude is 35 (upside down since the rider gets on at the bottom, so use a negative in the equation)

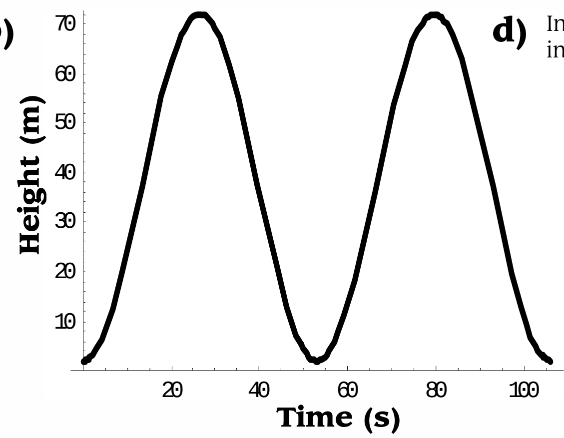
b-value: $b = \frac{2\pi}{P} = \frac{2\pi}{53}$

c-value: None for cosine

d-value: The lowest point is 2 m off the ground and the radius is 35 m, so the midline will be at 37 m.

$$h(t) = -35\cos\frac{2\pi}{53}t + 37$$

b)



c) To find the height at 81 seconds, plug the time into the equation.

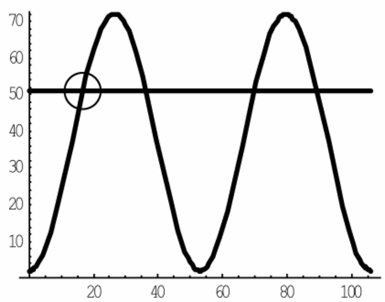
$$h(t) = -35\cos\frac{2\pi}{53}t + 37$$

$$h(81) = -35\cos\frac{2\pi}{53}(81) + 37$$

$$h(81) = 71.4$$

Or use the simpler method of 2nd → Trace → Value → x = 81

d) In your TI-83, graph the equation and also the line y = 51. The first intersection point will give the time the height first reaches 51 m.



51 m is first reached at 16.7 s

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2. a)

The diameter is 28 m, so the radius is 14.
The period is 15 seconds.

a-value: The amplitude is 14

b-value: $b = \frac{2\pi}{P} = \frac{2\pi}{15}$

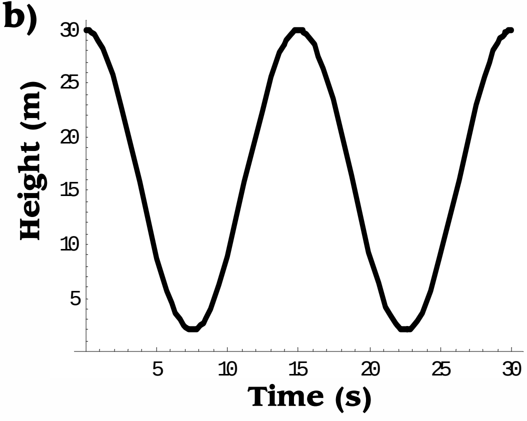
c-value: None for cosine

d-value: The highest point is 30 m off the ground and the radius is 14 m, so the midline will be at 16 m.

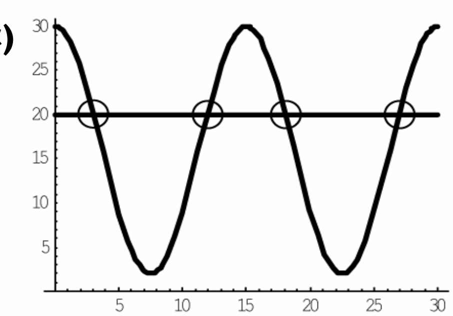
$$h(t) = 14\cos\frac{2\pi}{15}t + 16$$

*Use a positive since the rider gets on at the top, and therefore the cosine graph is upright.

b)



c)



The rider is at a height of 20 m at the following times:

3.06 s
11.9 s
18.1 s
26.9 s

3. The period is 1.46 seconds

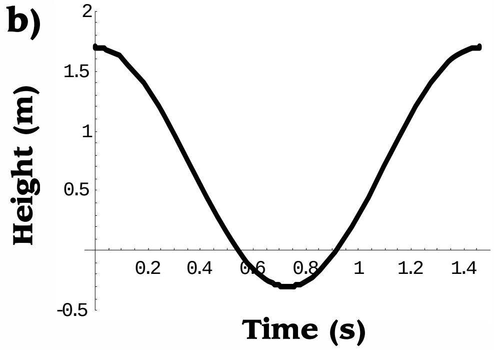
a) ***a-value:*** The amplitude is 1.

b-value: $b = \frac{2\pi}{P} = \frac{2\pi}{1.46}$

c-value: For the cosine graph, there is none.
d-value: The midline will be at 0.7 m, since the bottom is at -0.3 m and the radius is 1 m.

$$h(t) = \cos\frac{2\pi}{1.46}t + 0.7$$

b)



c)

Use your TI-83 to find the points of intersection with the line y=0. (In this case, it's the same as the x-intercepts)
The point on the wheel is first submerged at 0.545 s, and exits the water at 0.915 s. The total time underwater is 0.37 s.

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4. The period is 5 seconds.

a) ***a-value:*** The amplitude is 7.

$$amplitude = \frac{|\max - \min|}{2} = \frac{|22-8|}{2} = \frac{14}{2} = 7$$

b-value: $b = \frac{2\pi}{P} = \frac{2\pi}{5}$

c-value: For the cosine graph, there is none.

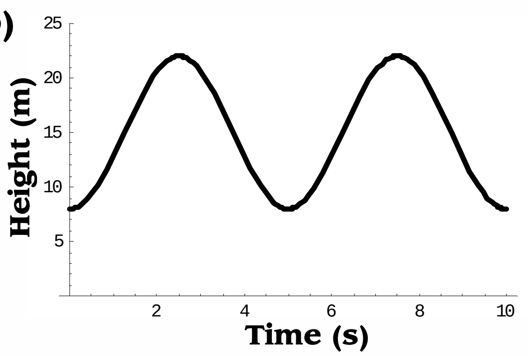
d-value: The midline is at 15.

$$midline = \frac{\max + \min}{2} = \frac{22+8}{2} = \frac{30}{2} = 15$$

$$h(t) = -7\cos\frac{2\pi}{5}t + 15$$

The cosine pattern is upside down, so we need a negative.

b)



c) To find the height at 4 seconds, plug the time into your equation.

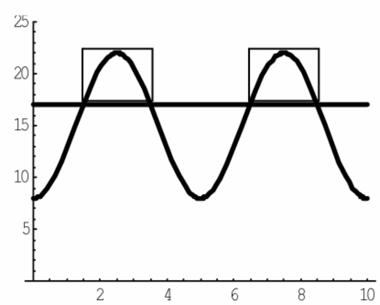
$$h(t) = -7\cos\frac{2\pi}{5}t + 15$$

$$h(4) = -7\cos\frac{2\pi}{5}(4) + 15$$

$$h(4) = 12.8\text{m}$$

Or, use 2nd → Trace → Value → x = 4

d)



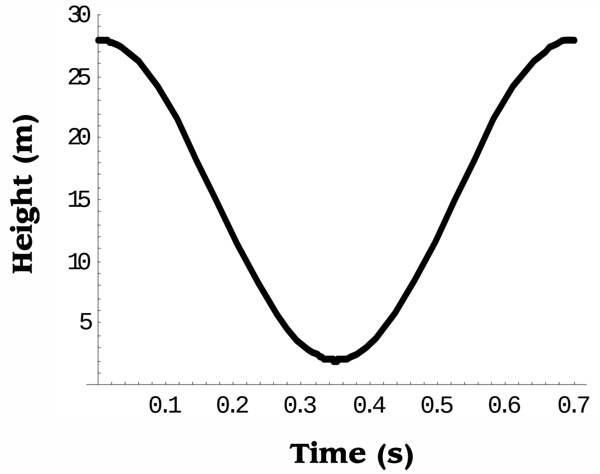
In the first cycle, the point is above 17 m between 1.48 s and 3.52 s, for a total of 2.04 seconds. Since the second cycle is identical to the first, the total time spent above 17 m is 4.08 s.

5. a) The midline is at 15 m, and the radius is 13 m, so the maximum height is 28 m.

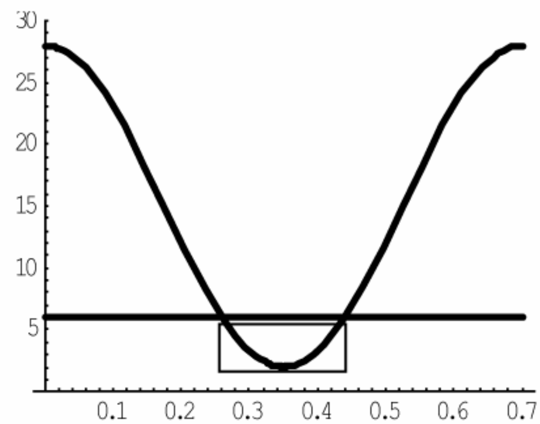
b) The maximum occurs at the beginning of the cycle (0 s) and the end (0.7 s)

c) The midline is at 15 m, and the radius is 13 m, so the minimum height is 2 m.

d) the minimum is reached exactly halfway, so 0.35 s.



e) The point first goes under 6 m at 0.26 s, and then rises above at 0.44 s. The total time then is 0.18 s.



f) We need to convert the time to seconds.
2 hours and 20 minutes is 140 min.
140 min = 8400 s.

$$h(t) = 13\cos\frac{2\pi}{0.7}t + 15$$

$$h(8400) = 13\cos\frac{2\pi}{0.7}(8400) + 15$$

$$h(8400) = 28$$

Events that are cyclic, such as seasonal variations in temperature, can be modeled with trigonometric functions.

Example: The average temperature for Regina is hottest at 27 °C on July 28, and coolest at -16 °C on January 10.
a) Write the cosine equation for the graph.
b) Draw the graph that approximates the temperature curve for the year.
c) What is the average temperature expected for October 4?
d) The average temperature is higher than 23 °C for how many days?

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a) Whenever there are questions that deal with values over an entire year, the period is 365.

a-value: The amplitude is 21.

$$amplitude = \frac{|\max - \min|}{2} = \frac{|27-(-16)|}{2} = \frac{43}{2} = 21.5$$

b-value: $b = \frac{2\pi}{P} = \frac{2\pi}{365}$

c-value: We are starting the graph on January 1, so that is day zero. The maximum value (where the positive cosine graph would begin) is on July 28.
So, the number of days to July 28 is:
31+28+31+30+31+30+28 = 209

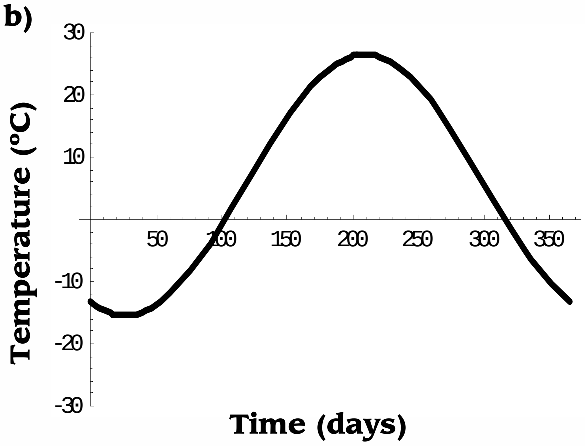
d-value: The midline is at 5.5.

$$midline = \frac{\max + \min}{2} = \frac{27+(-16)}{2} = \frac{11}{2} = 5.5$$

$$T(d) = 21.5\cos\frac{2\pi}{365}(d-209)+5.5$$

Where T is temperature in °C, and d is the number of days.

b)



c) To find the expected temperature on October 4, first find what day of the year it is.

31+28+31+30+31+30+31+31+30+4 = 277

Now use this in your equation:

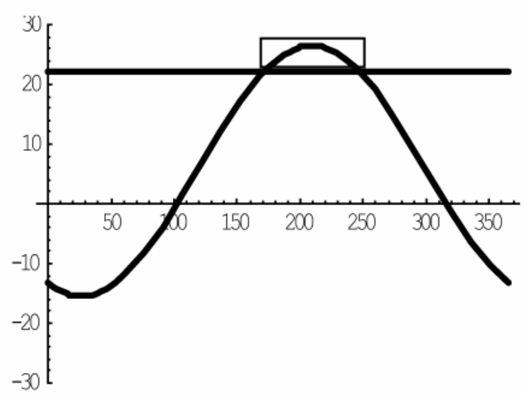
$$T(d) = 21.5\cos\frac{2\pi}{365}(d-209)+5.5$$

$$T(277) = 21.5\cos\frac{2\pi}{365}(277-209)+5.5$$

$$T(277) = 13.9$$

Or, graph in your TI-83 and type:
2^{nd} → Trace → Value → x = 277

d)



Use your TI-83 to find the intersection points of the graph and the line y=23.

The first day over 23 °C is day 173. The last day over 23 °C is day 245.
The total time is 72 days.

Trigonometry Lesson 8:
Part II – Year Questions

Example 2: **The earliest sunset occurs at 5:34 PM on Dec. 21, and the latest at 11:45 PM on June 21.**
a) Write the cosine equation of the graph.
b) Draw the graph approximating the sunrise times throughout the year.
c) What is the sunset time on April. 6?
d) The sunset time is earlier than 8 PM for what percentage of the year?

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a) Before anything else, we are never allowed to use time in hour/minute form when creating an equation! Convert to decimal hour form.

Conversion Reminder:

To get the hours, think of the 24-hour clock. Then divide the number of minutes by 60 to finish off the decimal.

5:34 PM → 5 PM is 17 hours on the 24-hour clock. Then, 11/60 = 0.57. So, the time we use is 17.57

11:45 PM → 11 PM is 23 hours on the 24-hour clock. Then, 45/60 = 0.75. So, the time we use is 23.75

More Examples: 12:00 AM = 0 hours
12:30 AM = 0.5 hours
7:15 PM = 19.25 hours
11: 49 PM = 23.82 hours

a-value: The amplitude is 3.1

$$amplitude = \frac{|\max - \min|}{2} = \frac{|23.75 - 17.57|}{2} = \frac{6.18}{2} = 3.1$$

b-value: $b = \frac{2\pi}{P} = \frac{2\pi}{365}$

c-value: We are starting the graph on January 1, so that is day zero. The maximum value (where the cosine graph would begin) is on June 21. So, the number of days is:
31+28+31+30+31+21 = 172

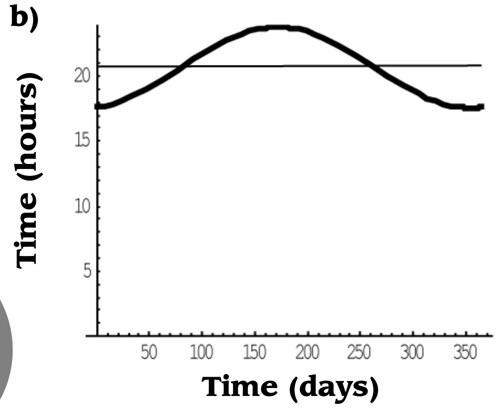
d-value: The midline is at

$$midline = \frac{\max + \min}{2} = \frac{23.75 + 17.57}{2} = \frac{41.32}{2} = 20.66$$

$$S(d) = 3.1\cos\frac{2\pi}{365}(d - 172) + 20.66$$

S is the time of sunset (in decimal hours)
d is the day number of the year.

b)



c) To find the expected temperature on April 6, first find what day of the year it is.

31+28+31+ 6 = 96

Now use this in your equation:

$$T(d) = 3.1\cos\frac{2\pi}{365}(d - 172) + 20.66$$

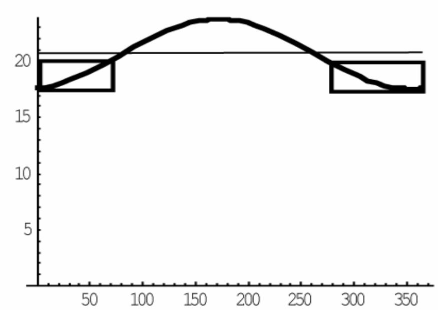
$$T(96) = 3.1\cos\frac{2\pi}{365}(96 - 172) + 20.66$$

$$T(96) = 21.46$$

21.46 = 9:28 PM

Or, graph in your TI-83 and type:
2nd → Trace → Value → x = 96

d) Sunset is earlier than 8 PM from day 0 to 68, then again from day 276 to 365. So, the total number of days is 157. Divide by the total days in the year to get 43%



1. The highest average temperature for the Edmonton region is 24 °C, and occurs on July 20. The coldest average temperature is -16 °C, and occurs on January 14.

a) Write a cosine equation for the graph.

b) Draw the graph that approximates the temperature curve for the year.

c) What is the average temperature expected for November 4?

d) The average temperature is below 0 °C for how many days?

2. The average temperature for the Ottawa region is hottest at 25 °C on July 23, and coolest at 4 °C on January 12.

a) Write a cosine equation for the graph.

b) Draw the graph that approximates the temperature curve for the year.

c) What is the average temperature expected for August 4?

d) The average temperature is higher than 20 °C for how many days?

3. The following table gives the average recorded monthly temperature throughout the year. ***(Use month number for the x-axis. eg. April is x = 4).***

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Temp	-17	-16	-9	1	10	16	22	20	15	11	2	-11

a) What is the amplitude *(Based on coldest & warmest months)* and period of the function?

b) Write a cosine equation for the graph.

c) Sketch the graph that approximates the temperature curve for the year.

d) How does the recorded temperature for September compare with the value from your equation?

**This question can also be completed using the trigonometric regression feature of your TI-83.*

Note, however, that trigonometric regression is not a requirement for this course.

4. The latest sunrise occurs at 9:10 AM on December 21. The earliest occurs at 3:43 AM on June 21.

a) Write the cosine equation of the graph.

b) Draw the graph approximating the sunrise times throughout the year

c) What is the sunrise time on Feb. 21?

d) The sunrise time is earlier than 5 AM for how many days?

5. The earliest sunset occurs at 5:14 PM on Dec. 21, and the latest at 11:54 PM on June 20.

a) Write the cosine equation of the graph.

b) Draw the sinusoidal graph approximating the sunrise times throughout the year.

c) What is the sunset time on April. 6?

d) The sunset time is earlier than 8 PM for what percentage of the year?

6. The following table gives the average recorded sunrise time for each month.
(Use month number for the x-axis. eg. April is x = 4)

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Time	8:30	7:59	7:15	6:30	6:01	5:30	5:34	6:03	6:34	7:10	7:52	8:25

a) What is the amplitude and period of the function?

b) Write a cosine equation for the graph.

c) Sketch the graph that approximates the sunrise times for the year.

d) How does the recorded sunrise time for April compare with the value from your equation?

Trigonometry Lesson 8:

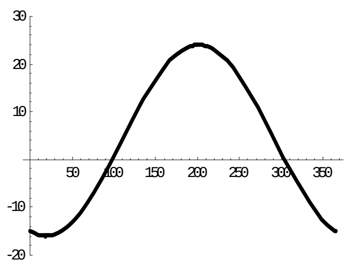
Part II – Year Questions **ANSWERS**

1.

a)

$$T(d) = 20\cos\frac{2\pi}{365}(d-201)+4$$

b)



c) -1.36 °C

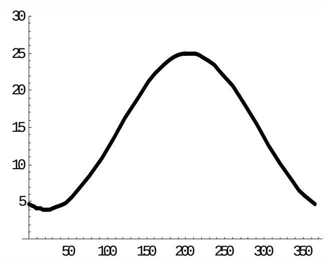
d) 159 days

2.

a)

$$T(d) = 10.5\cos\frac{2\pi}{365}(d-204)+14.5$$

b)



c) 24.8 °C

d) 118 days

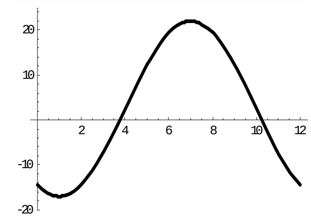
3.

a) Amplitude is $\frac{22-(-17)}{2} = 19.5$

Period is 12 months

b) $T(d) = 19.5\cos\frac{2\pi}{12}(d-7)+2.5$

c)

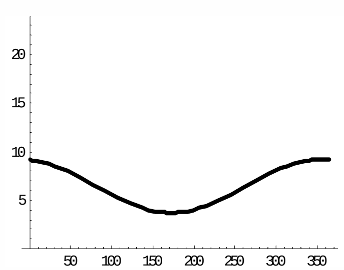


d) 12.3 °C is the answer from the equation, which is slightly off from the recorded value of 15 °C.
This is because the equation is an approximation of the actual temperatures.

4.

a) $T(d) = -2.73\cos\frac{2\pi}{365}(d-172)+6.45$

b)



c) 7.74 hours → 7:44 AM

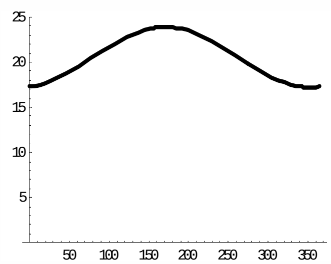
d) 118 days

5.

a)

$$T(d) = 3.34\cos\frac{2\pi}{365}(d-171)+20.57$$

b)



c) 21.50 hours → 9:30 PM

d) 163 days

6.

*Use decimal time

Latest sunset = 8.50 hours

Earliest sunset = 5.50 hours

a) Amplitude is $\frac{8.5-5.5}{2} = 1.5$

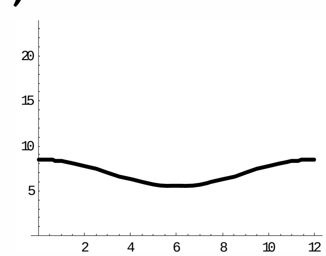
Period is 12

Midline is $\frac{8.5+5.5}{2} = 7$

b) $T(d) = -1.5\cos\frac{2\pi}{12}(d-6)+7$

Use a negative for the cosine pattern since it is upside down.

c)



d) From equation → 6:15 AM

From given data → 6:30 AM

The time from the equation is slightly off from the recorded time, but that is to be expected since the equation is only an approximation of the actual curve.