Pure Math 30:

TRIGONOMETRY 1

$90° = \dfrac{\pi}{2}$
$\{0,1\}$

$120° = \dfrac{2\pi}{3}$ $\left\{-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right\}$

$60° = \dfrac{\pi}{3}$ $\left\{\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right\}$

$135° = \dfrac{3\pi}{4}$ $\left\{-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right\}$

$45° = \dfrac{\pi}{4}$ $\left\{\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right\}$

$150° = \dfrac{5\pi}{6}$ $\left\{-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right\}$

$30° = \dfrac{\pi}{6}$ $\left\{\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right\}$

$180 = \pi$ $\{-1,0\}$

$0° = 360° = 2\pi$ $\{1,0\}$

$210° = \dfrac{7\pi}{6}$ $\left\{-\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right\}$

$330° = \dfrac{11\pi}{6}$ $\left\{\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right\}$

$225° = \dfrac{5\pi}{4}$ $\left\{-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right\}$

$315° = \dfrac{7\pi}{4}$ $\left\{\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right\}$

$240° = \dfrac{4\pi}{3}$ $\left\{-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right\}$

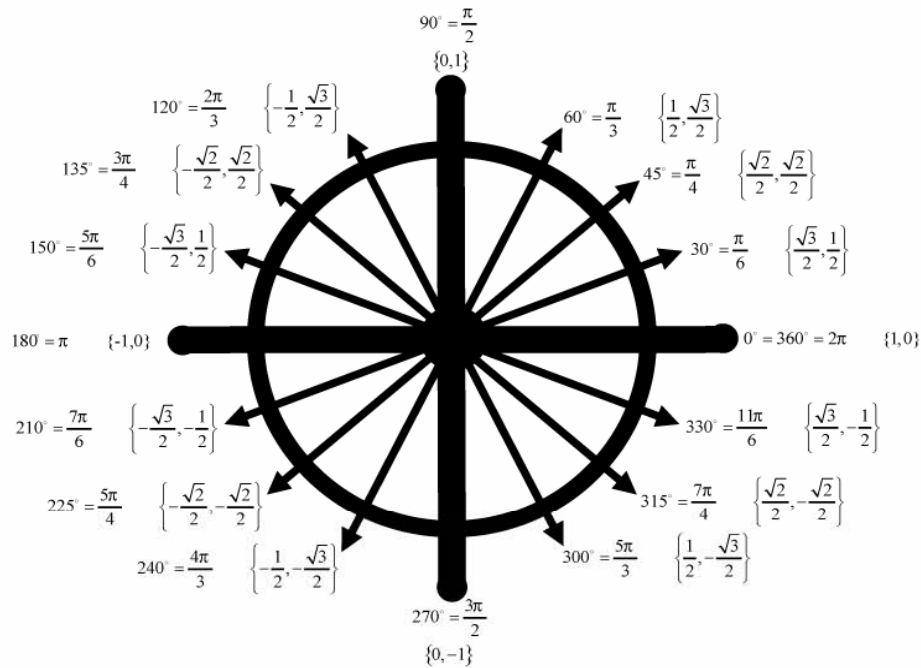$300° = \dfrac{5\pi}{3}$ $\left\{\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right\}$

$270° = \dfrac{3\pi}{2}$
$\{0,-1\}$

Lesson Seven
Radian Graphs

Pure Math 30: EXPLAINED!

By Barry Mabillard

TRIGONOMETRY LESSON SEVEN
Part I   Graphing Radian Functions

In application questions, when your x-axis is time, distance, or some other unit, you must do your trig graph in radian mode. Basically, whenever you see integers on the x-axis, (rather than degrees or radian fractions) you need to be in radian mode.

In the graph on the right, notice how the variable is time instead of θ. This is a clue you need radian mode.

$$y = 1.23\cos\frac{2\pi}{12.1}t + 1.85$$
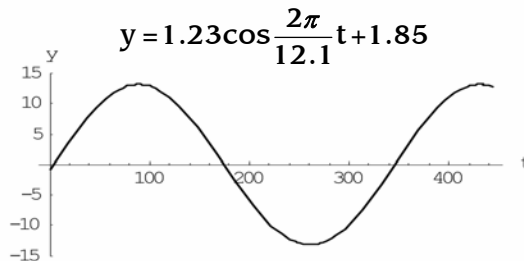
Example 1: Given the equation: $y = 13.2\cos\frac{2\pi}{342}(t-101)+6.5$ find appropriate units for your window and graph with the TI-83.

1) First you must figure out the period so you know how long a cycle is.

$$Period = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{342}} = 2\pi \times \frac{342}{2\pi} = 342$$

The length of one complete cosine cycle is 342 units.
Also, since the pattern starts at 101 due to the phase shift, we want to see all the way to 443 (phase shift + period) for the complete picture.

2) Next you need to know where the minimum and maximum values are using the following formulas:

Minimum = d – a
Minimum = 6.5 – 13.2
Minimum = -6.7

Maximum = d + a
Maximum = 6.5 + 13.2
Maximum = 19.7

3) Now choose a scale. The best way to do this is by picking numbers large enough that you won't have too many ticks on either axis. For the x-axis, we're going from 0 to about 450, so use a scale of 100. For the y-axis, since we want to see from -7 to +20, a scaling of 5 would be good. Of course, these are just guidelines and you could use several different scales and still obtain a good graph.

4) Use the following window settings:

X_{min}: 0
X_{max}: 443
X_{scl}: 20
Y_{min}: -6.7
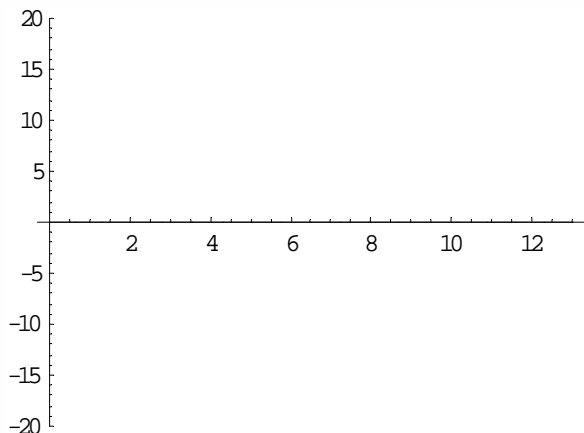Y_{max}: 19.7
Y_{scl}: 5
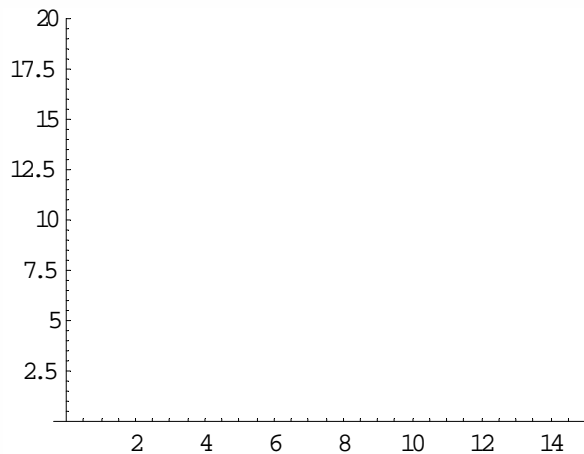
5) Now draw the graph:
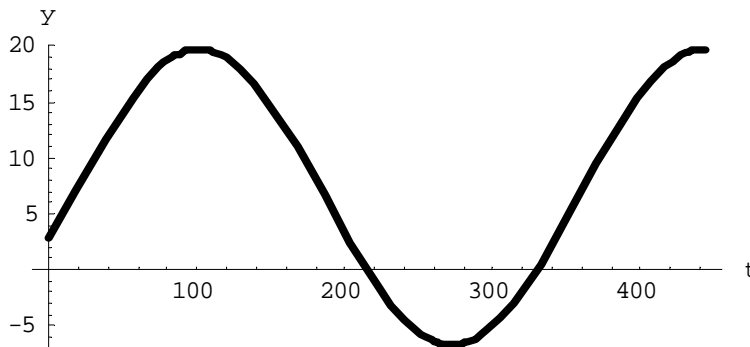
TRIGONOMETRY LESSON SEVEN
Part I   Graphing Radian Functions

Questions: For each of the following equations, find appropriate window settings.
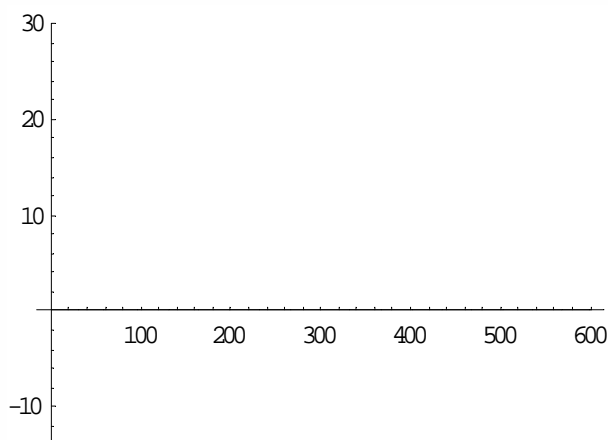Then draw the graph using your TI-83.

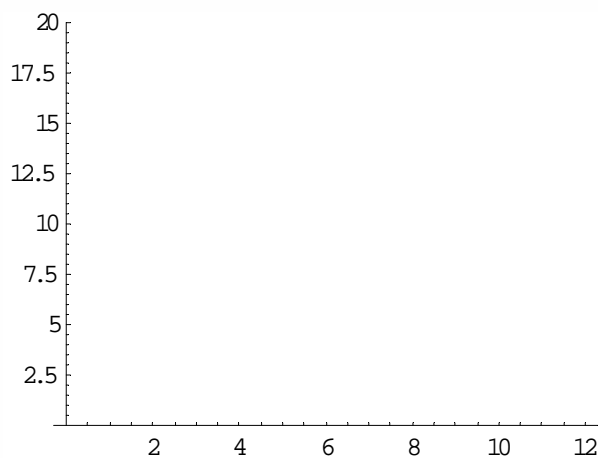**1.** $y = 17.2\cos\frac{\pi}{3}(t-7)+0.5$

2. $y = 2.2\sin 0.123\pi(t+1.7)+15.2$

3. $y = 20.1\sin\frac{2\pi}{300}(t-265)+6.2$

4. $y = -3.2\cos 0.18\pi t+17$

Answers: **1.**

1) Find the period: $Period = \dfrac{2\pi}{b} = \dfrac{2\pi}{\dfrac{\pi}{3}} = 2\pi \times \dfrac{3}{\pi} = 6$

Since the cosine pattern starts at 7 due to the phase shift, and the period is 6, we want to see up to 13 on the x-axis.

2) Find the minimum and maximum:
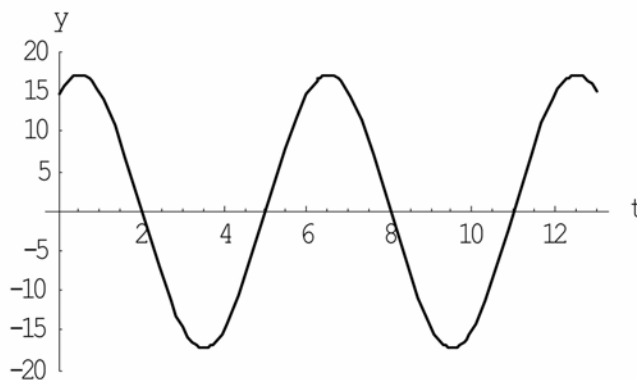*Minimum* = d – a = 0.5 – 17.2 = -16.7
Maximum = d + a = 0.5 + 17.2 = 17.7

3) Choose a scaling, 2 for the x-axis and 5 for the y-axis will work fine.

4) Use the following window settings:

X_{min}: **0**
X_{max}: **13**
X_{scl}: **2**
Y_{min}: **-16.7**
Y_{max}: **17.7**
Y_{scl}: **5**

5) Draw the graph.

2.

1) Find the period: $Period = \dfrac{2\pi}{b} = \dfrac{2\pi}{0.123\pi} = 16.23$

Since the sine pattern starts at -1.7 due to the phase shift, and the period is 16.23, we want to see at least up to 14.53 on the x-axis for the complete picture.

2) Find the minimum and maximum:
Minimum = d – a = 15.2 – 2.2 = 13
Maximum = d + a = 15.2 + 2.2 = 17.4

**3)** Choose a scaling: 2 for the x-axis and 5 for the y-axis will work fine.

4) Use the following window settings

X_{min}: **0**
X_{max}: **14.5**
X_{scl}: **2**
Y_{min}: **0**
Y_{max}: **17.4**
Y_{scl}: **2.5**

We have two options for where to set the Y_{min} value.

If we use 13, we'll get a big display of the graph, but lose the frame of reference with the origin.

If we use 0, we'll keep our frame of reference (which can be very useful in applications), but have a small display.
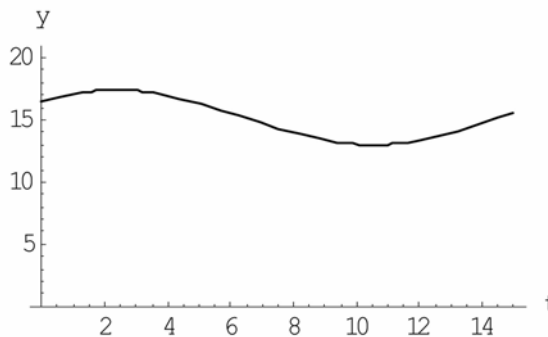
TRIGONOMETRY LESSON SEVEN

Part 1   Graphing Radian Functions

Answers: 3.

1) Find the period: $Period = \dfrac{2\pi}{b} = \dfrac{2\pi}{\dfrac{2\pi}{300}} = 2\pi \times \dfrac{300}{2\pi} = 300$

Since the sine pattern starts at 265 and the period is 300, we should extend our *x*-axis to 565.

2) Find the minimum and maximum:
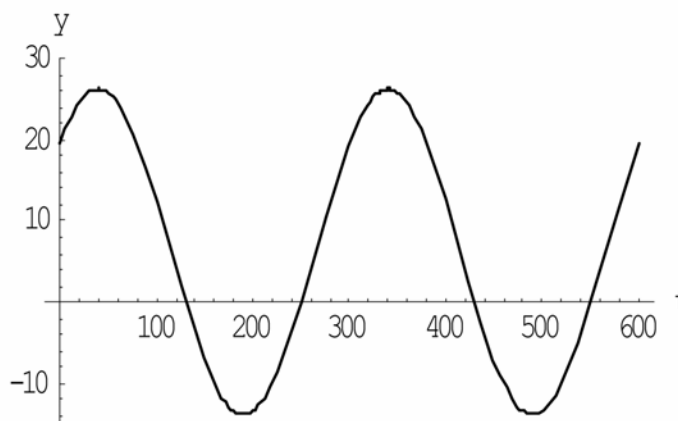*Minimum* = d – a = 6.2 – 20.1 = -13.9
Maximum = d + a = 6.2 + 20.1 = 26.3

5) Graph the equation

3) Choose a scaling: 100 for the x-axis, and 10 for the y-axis.

4) Use the following window settings

X_{min}: **0**
X_{max}: **565**
X_{scl}: **100**
Y_{min}: **-14**
Y_{max}: **27**
Y_{scl}: **10**

4.

1) Find the period: $Period = \dfrac{2\pi}{b} = \dfrac{2\pi}{0.18\pi} = 11.11$

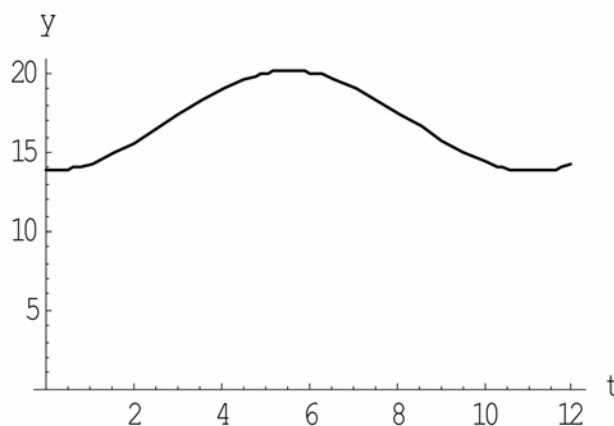**2)** Find the minimum and maximum:
Minimum = d – a = 17 – 3.2 = 13.8
Maximum = d + a = 17 + 3.2 = 20.2

5) Graph the equation

3) Choose a scaling, 2 for the x-axis and 5 for the y-axis.

4) Use the following window settings

X_{min}: **0**
X_{max}: **11**
X_{scl}: **2**
Y_{min}: **0**
Y_{max}: **27**
Y_{scl}: **5**

PURE MATH 30: EXPLAINED!

Example 1: The height of an object is given by the equation:

$h(t) = 2.2\sin 0.123\pi(t+1.7)+15.2$

Find the height after 2.4 seconds.

We may evaluate a point on a trig function by using the TI-83.

$h(2.4) = 2.2\sin 0.123\pi(2.4+1.7)+15.2$ **(Radian Mode)**

$h(2.4) = 17.4$

Alternatively, we could draw the graph and use:
2nd→ Trace
→ Value
→ x = 2.4

Questions: Evaluate the following functions for the value indicated.

1) Evaluate $h(t) = 13.2\cos\dfrac{2\pi}{342}(t-101)+6.5$ when t = 105.

2) Evaluate $h(t) = 20.1\sin\dfrac{2\pi}{300}(t-265)+6.2$ when t = 296

3) Evaluate $h(t) = 18.5\cos\dfrac{2\pi}{365}(t-28)+4.5$ when t = 45

4) Evaluate $h(t) = 12\sin\dfrac{\pi}{2}(t-3)-3$ when t = 4
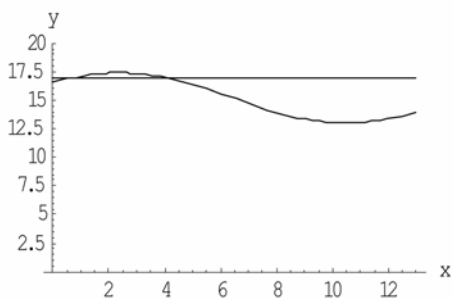
Answers:

1) 19.7

2) 18.4

3) 22.2

4) 9

Sometimes the y-variable is given, and we need to find the x-variable. In this case, we must graph both the left side and right side of the equation, then find the intersection point.

Example 1: The height of an object is given by the equation: $h(t) = 2.2\sin0.123\pi(t+1.7)+15.2$
Find the time when the object first reaches a height of 17.

In your graphing calculator, graph:
$y_1 = 2.2\sin0.123\pi(t+1.7)+15.2$
$y_2 = 17$

Now find the first intersection point.

TI-83: 2nd →Trace→Intersect

The *x*-value of the first intersection point will be the time the object first reaches a height of 17.

Questions: Find the time when each of the following heights are reached. Don't forget to set your window properly!

1) Find the time when a height of 12 is first reached in the equation:

$$h(t) = 13.2\cos\frac{2\pi}{342}(t-101)+6.5$$

2) Find the time when a height of 23 is first reached in the equation:

$$h(t) = 20.1\sin\frac{2\pi}{300}(t-265)+6.2$$

3) Find the time when a height of 12 is first reached in the equation:

$$h(t) = 18.5\cos\frac{2\pi}{365}(t-28)+4.5$$

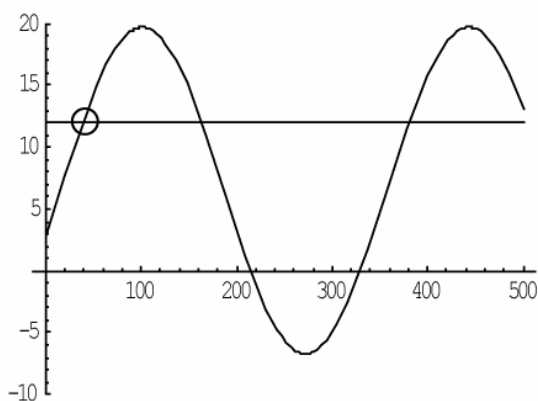**4)** Find the time when a height of 0 is first reached in the equation:

$$h(t) = 12\sin\frac{\pi}{2}(t-3)-3$$

Answers:

1)

t = 38.9 s

2)

t = 12.3 s

3)

t = 95 s

4)

t = 0.84 s