COURSE REVIEW

Transformations:

1) State the transformation(s) that each function has undergone. Then state the transformations in function and in mapping notation.

a) $y = -\frac{1}{4}\sqrt{3(x-7)}$, base function: $f(x) = \sqrt{x}$

V. compression by a factor of 4

reflection along the x-axis

H. compression by a factor of 3

H. translation to the right 7 units

$y = \frac{-1}{2} f(3(x-7))$

$(x, y) \rightarrow (\frac{1}{3}x+7, \frac{-1}{2}y)$

b) $y = 3(4-x)^3 - 6$, base function: $f(x) = x^3$

$y = 3(-x+4)^3 - 6$

$y = 3[-1(x-4)]^3 - 6$

V. stretch by a factor of 3

reflection along the y-axis.

H. shift right 4 units

V. translation 6 units down

$y = 3f(-(x-4)) - 6$

$(x, y) \rightarrow (-x+4, 3y-6)$

c) $y = -3(4)^{x-1}$, compare to $f(x) = (4)^x$

reflection along the x-axis

V. stretch by a factor of 3

H. translation right 1 unit

fnct. notation: $y = -3f(x-1)$

mapping notation: $(x, y) \rightarrow (x+1, -3y)$

d) $y = \log(-x)$, compare to $f(x) = \log(x)$

reflection along the y-axis

fnct. notation: $y = f(-x)$

mapping notation: $(x, y) \rightarrow (-x, y)$

e) $y=-2\cos\frac{1}{3}\left(\theta-\frac{\pi}{2}\right)+1$, compare to $f(\theta)=\cos(\theta)$

reflection along the x-axis

V. stretch by a factor of 2

H. stretch by a factor of 3

V. shift up 1 unit

fnct. notation: $y=-2f\left(\frac{1}{3}\left(\theta-\frac{\pi}{2}\right)\right)+1$

mapping notation: $(x,y)\rightarrow\left(3x+\frac{\pi}{2},\ -2y+1\right)$

2) The graph of $f(x)=x^4$ is horizontally stretched by a factor of 2, reflected in the *y*-axis, and shifted up 5 units. Find the equation of the transformed function.

H. stretch by a factor of 2

reflected in the y-axis

shifted up 5 units

$y=\left(-\frac{1}{2}(x)\right)^4+5$

$y=\left(-\frac{1}{2}x\right)^4+5$

note this would be the same graph as $y=\frac{1}{16}x^4+5$

Inverses:

3) Determine the inverse of each of the following. State if the invese is not a function and state any restrictions.

a) $y=x^2-10$

inverse: $x=y^2-10$

$x+10=y^2$

$\pm\sqrt{x+10}=y$

not a fnct.

$D_{\text{inverse}}:\{x\mid x\geq -10,\ x\in\mathbb{R}\}$

extra: for the inverse to be a fnct $D_{\text{original}}:\{x\mid x\geq 0,\ x\in\mathbb{R}\}$ or $x\leq 0$

b) $f(x)=2x+1$

f: $y=2x+1$

inverse: $x=2y+1$

$x-1=2y$

$\frac{1}{2}x-\frac{1}{2}=y$

$\frac{1}{2}x-\frac{1}{2}=f^{-1}(x)$

$\uparrow$

$D_{f^{-1}}:\{x\mid x\in\mathbb{R}\}$ $\therefore$ inverse is a fnct.

c) $y = \frac{1}{x+3} - 1$

inverse: $x = \frac{1}{y+3} - 1$

$x + 1 = \frac{1}{y+3}$

$y + 3 = \frac{1}{x+1}$

$y = \frac{1}{x+1} - 3$

$f^{-1}(x) = \frac{1}{x+1} - 3$

$D_{f^{-1}(x)} : \{x \mid x \neq -1, x \in \mathbb{R}\}$

d) $y = \log_2 x$

inverse: $x = \log_2 y$

$\Leftrightarrow \quad 2^x = y$

$2^x = f^{-1}(x)$

$D_{f^{-1}} : \{x \mid x \in \mathbb{R}\}$

e) $y = 10^{x-4} + 7$

inverse: $x = 10^{y-4} + 7$

$x - 7 = 10^{y-4}$

$\Leftrightarrow \quad y - 4 = \log_{10}(x-7)$

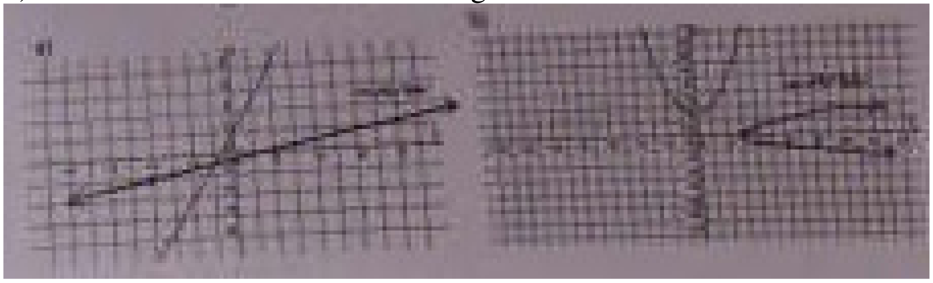
$y = \log_{10}(x-7) + 4$

$f^{-1}(x) = \log_{10}(x-7) + 4$

The inverse is a fnct!

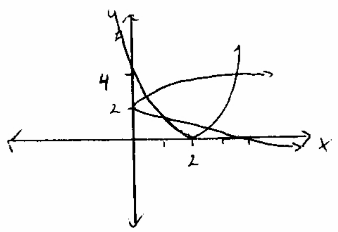
$D_{f^{-1}} : \{x \mid x > 7, x \in \mathbb{R}\}$

4) Sketch the inverse of the following functions on the same axes.



5) Sketch $f(x) = (x-2)^2$ and its inverse. What would the domain have to be so that the inverse is a function?

$f(x) = (x-2)^2$



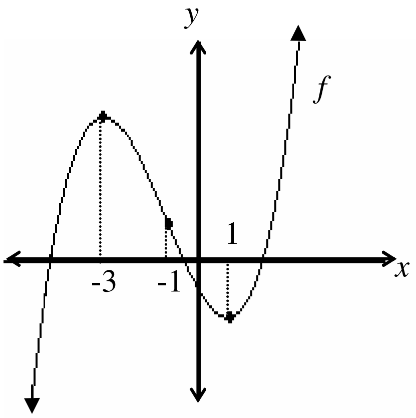
$D_f : \{x \mid x \geq 0, x \in \mathbb{R}\}$
so that inverse is a fnct.

$D_f : \{x \mid x \leq 0, x \in \mathbb{R}\}$
so that inverse is a fnct.

Functions, properties and their graphs:

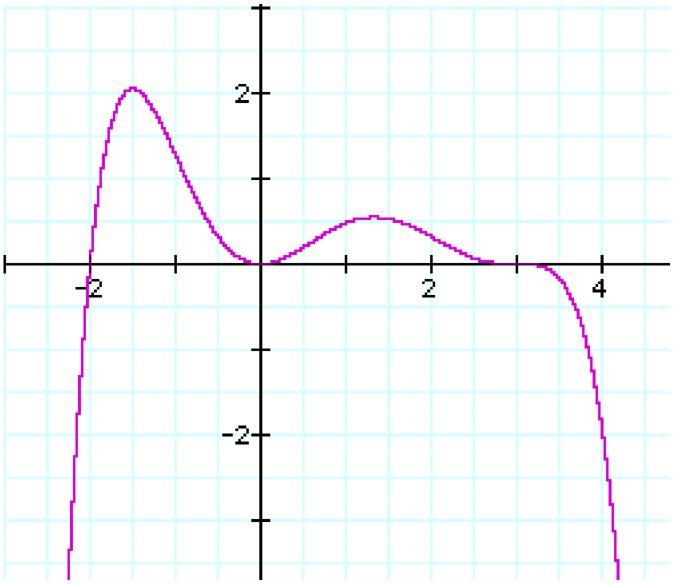
6) Given the graph of $y = f(x)$ shown on the right, state the intervals of x for which
(a) the function is decreasing

(b) the function is concave up



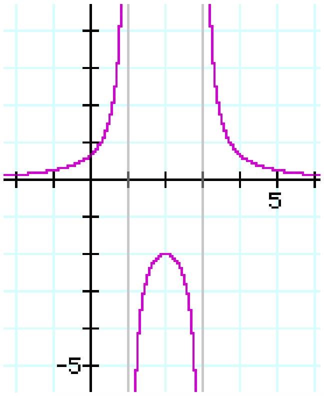
a) fnct is decr.: $-3 < x < 1$
b) fnct is conc. down: $x < -1$

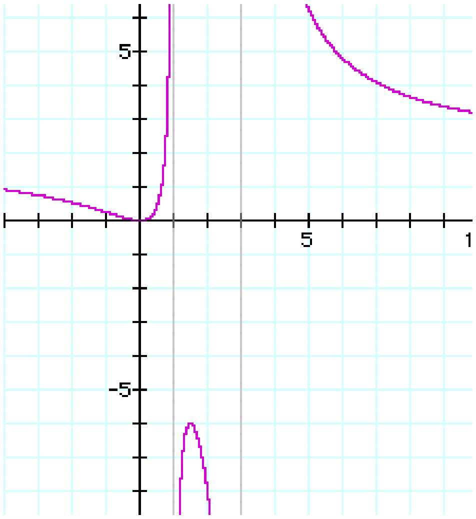
7) For the following graph, state the number of turning points, the number of inflection points, and the intervals of increase and decrease:



3 turning pts (2 max and one min.)
4 points of inflections
incr. $x < -1.5$ or $0 < x < 1.3$
decr. $-1.5 < x < 0$ or $1.3 < x < 3$ or $x > 3$

8) Which graph best matches the equation: $y = \dfrac{2x^2 + x - 3}{x^2 - 4x + 3}$?





8) $y = \dfrac{2x^2 + x - 3}{x^2 - 4x + 3}$

$P = -6$
$S = 1$ $3, -2$

$P = 3$
$S = -4$ $-3, -1$

factor to determine zeros, VA, holes, HA/OA

$y = \dfrac{(2x+3)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}}$

$y = \dfrac{2x+3}{x-3}$; $x \neq 1$

$\therefore$ hole at $x = 1$

let $x = 1$

$y = \dfrac{2(1)+3}{1-3}$

$y = -\dfrac{5}{2}$

$\therefore$ hole $(1, \frac{-5}{2})$

VA: let $x - 3 = 0$

$x = 3$

zero: let $2x + 3 = 0$

$2x = -3$

$x = \dfrac{-3}{2}$

HA: equal degrees
$\therefore$ divide coeff.

$y = 2$

These properties match the 3rd graph.

9) Sketch each of the following functions labelling: intercepts and asymptotes and stating the domain.

a) $y = -(x+2)^3$

→ special cubic, no asymptotes.

→ -ve lead coeff

∴ as $x \to \infty$, $y \to -\infty$

→ shifted left 2 units

→ zero $^-2$, pt. of inflection

x-int (let $y=0$)

$0 = -(x+2)^3$

$0 = (x+2)^3$

$\sqrt[3]{0} = x+2$

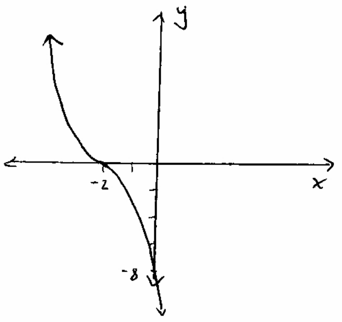
$0 = x+2$

$-2 = x$

y-int. (let $x=0$)

$y = -(2)^3$

$y = -8$



D: $\{x \mid x \in \mathbb{R}\}$

b) $y = (x-3)^2(x^2-4x-7)$

→ quartic, no asymptotes

→ +ve lead coeff

∴ as $x \to \infty$, $y \to \infty$

→ DR at $x=3$, turning pt.

x-int (let $y=0$)

$(x-3)^2=0$

$x=3$

$x^2-4x-7=0$

$x = \frac{4 \pm \sqrt{16+28}}{2}$

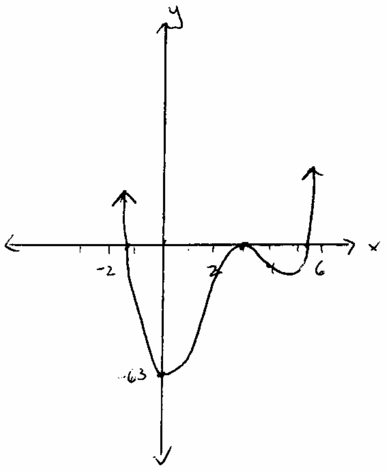
$x \doteq 5.32$

$x \doteq -1.32$

y-int (let $x=0$)

$y = (-3)^2(-7)$

$y = -63$



D:$\{x \mid x \in \mathbb{R}\}$

c) $y = x^4 + 2x^3 + x^2 + 2x$

→ quartic, no asymptotes

→ +ve lead coeff.

∴ as $x \to \infty$, $y \to \infty$

x-int. (let $y=0$)

$0 = x(x^3+2x^2+x+2)$

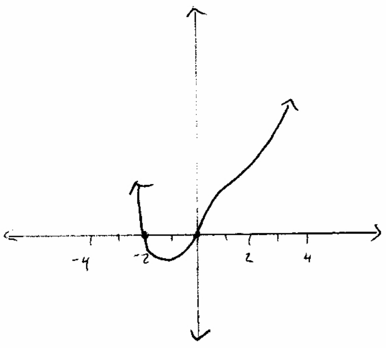
$0 = x[x^2(x+2)+1(x+2)]$

$0 = x(x+2)(x^2+1)$

$x=0$, $x=-2$ no real sol^n

y-int (let $x=0$)

$y=0$



D: $\{x \mid x \in \mathbb{R}\}$

d) $f(x)=\frac{x^2+3}{x+4}$

d) $f(x) = \frac{x^2+3}{x+4}$

→ rational fnct

→ deg numerator > deg denom. by 1 ∴ OA

→ +ve lead. coeff. ∴ ends in QI

x-int (let y=0)

$0 = \frac{x^2+3}{x+4}$

⇒ numerator = 0

$x^2+3=0$

no real solⁿ

∴ no x-int

y-int (let x=0)

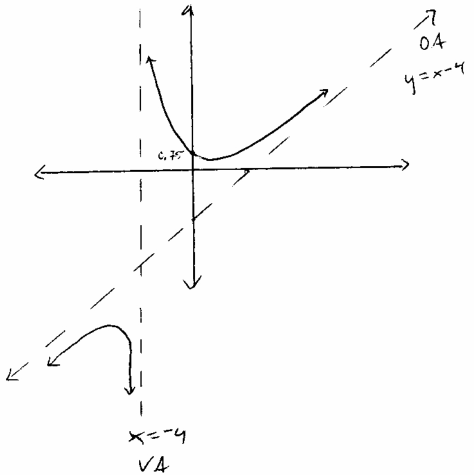
$y=\frac{3}{4}$

VA: $x=-4$

OA:

$$\begin{array}{r|rrr} -4 & 1 & 0 & 3 \\ & \downarrow & -4 & 16 \\ \hline & 1 & -4 & 19 \neq 0 \end{array}$$ ☺

O.A. $y = x-4$



D: $\{x \mid x \neq -4, x \in \mathbb{R}\}$

e) $f(x)=\log(x+2)$

9e) $f(x) = \log(x+2)$

→ logarithmic fnct, VA

→ shifted left 2 units

x-int (let y=0)

$0 = \log(x+2)$

⇔ $10^0 = x+2$

$1 = x+2$

$-1 = x$

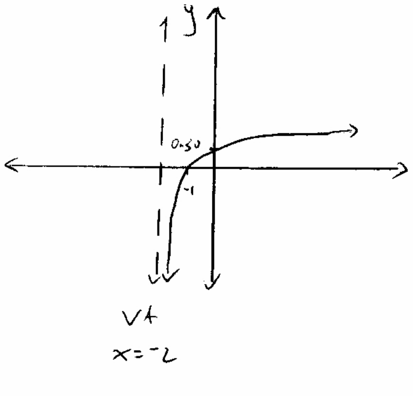
y-int (let x=0)

$y = \log 2$

$y \doteq 0.3010$

VA let $x+2=0$

$x=-2$



D: $\{x \mid x > -2, x \in \mathbb{R}\}$

f) $f(x) = 3 \cdot 5^{-x} + 4$

→ exponential fnct, HA

→ v. stretch by a factor of 3

→ reflection along y-axis

→ v. shift up 4 units

x-int (let $y=0$)

$0 = 3 \cdot 5^{-x} + 4$

$-4 = 3 \cdot 5^{-x}$

$-\frac{4}{3} = 5^{-x}$ (no sol^{n})

$\Leftrightarrow \log_5 \frac{-4}{3} = -x$ ← check

$\frac{\log \frac{-4}{3}}{\log 5} = -x$

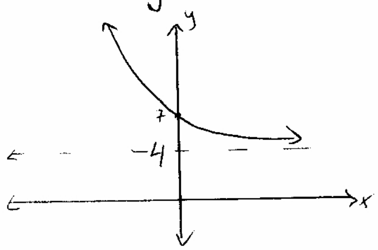
no sol$^{\underline{n}}$

y-int (let $x=0$)

$f(0) = 3 \cdot 5^{0} + 4$

$f(0) = 7$

HA $y = 4$



D: $\{x \mid x \in \mathbb{R}\}$

g) $f(x) = \dfrac{10 - 10x}{(x-4)^2}$

→ rational fnct

→ deg numerator < deg of denom
∴ HA

→ −ve lead coeff
∴ ends in QIV

x-int

let $-10x + 10 = 0$

$10 = 10x$

$1 = x$

y-int. let $x=0$

$f(0) = \frac{10}{(-4)^2}$

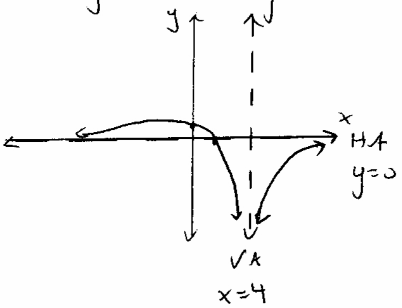
$f(0) = \frac{5}{8}$

VA let $(x-4)^2 = 0$

$x = 4$ DR

∴ arrows in same direction

HA $y = 0$ ∵ degrees



D: $\{x \mid x \neq 4, x \in \mathbb{R}\}$

h) $f(x) = \dfrac{x^2+4}{x^2-4}$

h) $f(x) = \dfrac{x^2+4}{x^2-4}$

→ rational fnct

→ deg numerator = deg denominator
∴ HA

→ +ve lead. coeff.
∴ ends Q I

x-int let $x^2+4=0$
no sol^{n}

y-int let $x=0$

$f(0) = \dfrac{4}{-4}$

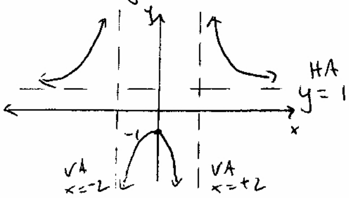
$f(0) = -1$

VA let $x^2-4=0$

$x = \pm 2$

HA ÷ coeff ∵ deg. equal

$y = 1$



i) $y = 2\sin\left(\theta - \dfrac{\pi}{2}\right) + 1$

→ sine fnct

→ Amp : 2

P.S : $\frac{\pi}{2}$

V. disp = 1

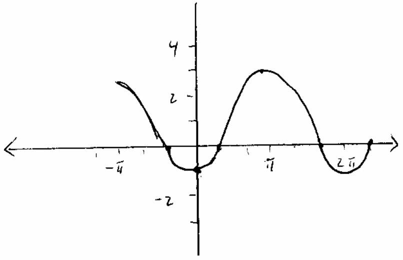
Period = 2π

y-int (let $\theta = 0$)

$y = 2\sin(-\frac{\pi}{2}) + 1$

$y = -2+1$

$y = -1$



D: $\{x \mid x \in \mathbb{R}\}$

x-int (let $y=0$)

$0 = 2\sin(\theta - \frac{\pi}{2}) + 1$

$-1 = 2\sin(\theta - \frac{\pi}{2})$

$\frac{-1}{2} = \sin(\theta - \frac{\pi}{2})$

let $B = \theta - \frac{\pi}{2}$

$\frac{1}{2} = \sin B_r$

$\frac{\pi}{6} = B_r$

$B_1 = \pi + \frac{\pi}{6}$ $\quad$ $B_2 = 2\pi - \frac{\pi}{6}$

$\theta_1 - \frac{\pi}{2} = \frac{7\pi}{6}$ $\quad$ $\theta_2 - \frac{\pi}{2} = \frac{11\pi}{6}$

$\theta_1 = \frac{7\pi}{6} + \frac{\pi}{2}$ $\quad$ $\theta_2 = \frac{11\pi}{6} + \frac{\pi}{2}$

$\theta_1 = \frac{10\pi}{6}$ $\quad$ $\theta_2 = \frac{14\pi}{6}$

$\theta_1 = \frac{5\pi}{3}$ $\quad$ $\theta_2 = \frac{7\pi}{3}$

$\theta_3 = \frac{5\pi}{3} \pm 2n\pi$ $\quad$ $\theta_4 = \frac{7\pi}{3} \pm 2n\pi$

j) $y = \tan 2\theta - 1$

j) $y = \tan 2\theta - 1$

→ tangent fnct, VA

→ Period $= \frac{\pi}{2}$

v. shift down 1

y-int (let $\theta = 0$)

$y = \tan 0 - 1$

$y = -1$

x-int (let $y = 0$)

$0 = \tan 2\theta - 1$

$1 = \tan 2\theta$

let $B = 2\theta$

$1 = \tan B$

$\frac{\pi}{4} = B_1 \qquad B_2 = \frac{5\pi}{4}$

$\frac{\pi}{8} = \theta_1 \qquad \theta_2 = \frac{5\pi}{8}$

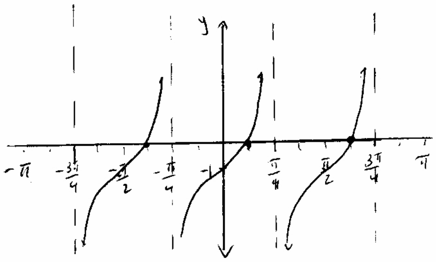
$\frac{\pi}{8} \pm \frac{n\pi}{2} = \theta_3 \qquad \theta_4 = \frac{5\pi}{8} \pm \frac{n\pi}{2}$

VA: for $y = \tan\theta$

VA $x = \frac{\pi}{2}$

H compression

$x = \frac{\pi}{4}$



D: $\{x \mid x \neq \frac{\pi}{4} + n\pi,\ n \in \mathbb{Z},\ x \in \mathbb{R}\}$

k) $y = 0.5\sec\left(\theta + \frac{\pi}{4}\right)$

→ secant fnct.

→ reciprocal of cosine

→ Amp not applicable

H. shift left $\frac{\pi}{4}$

y-int (let $\theta = 0$)

$y = 0.5 \sec\left(\frac{\pi}{4}\right)$

$y = 0.5 \cdot \frac{1}{\cos\frac{\pi}{4}}$

$y = 0.5 \cdot \frac{\sqrt{2}}{1}$

$y = \frac{\sqrt{2}}{2}$

$y \doteq 0.7071$

x-int (let $y = 0$)

$0 = 0.5 \sec\left(\theta + \frac{\pi}{4}\right)$

$0 = \sec\left(\theta + \frac{\pi}{4}\right)$

$0 = \frac{1}{\cos\left(\theta + \frac{\pi}{4}\right)}$

$\cos\left(\theta + \frac{\pi}{4}\right) =$ undef.

not possible.

VA for $y = \sec\theta$

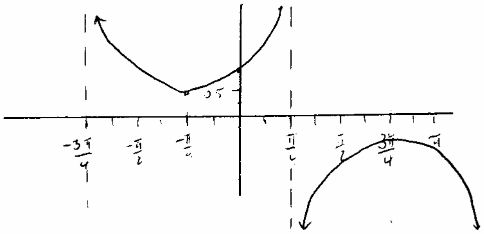
when $\theta = \frac{\pi}{2} \pm n\pi$

(zeros of $y = \cos\theta$)

H. shift left $\frac{\pi}{4}$

∴ VA $\theta = \frac{\pi}{2} - \frac{\pi}{4} \pm n\pi$

$\theta = \frac{\pi}{4} \pm n\pi$



D: $\{x \mid x \neq \frac{\pi}{4} \pm n\pi,\ n \in \mathbb{Z},\ x \in \mathbb{R}\}$

10) Analyze and sketch the following, using intercepts, asymptotes, and end behaviours:

$$y=\frac{3x^3+10x^2+3x}{x^2+5x+6}$$

zeros: let $3x^3+10x^2+3x=0$

$x(3x^2+10x+3)=0$

$x(3x+1)(x+3)=0$

$x=0,\ x=\frac{-1}{3},\ x=-3$

y-int let $x=0$

$y=\frac{0}{6}$

$y=0$

O.A.: $x^2+5x+6 \overline{)3x^3+10x^2+3x+0}$ quotient $3x-5$

$-(3x^3+15x^2)$

$-5x^2+3x+0$

$-(-5x^2-25x-30)$

$28x+30$

O.A. $y=3x-5$

VA let $x^2+5x+6=0$

$(x+3)(x+2)=0$

$x=-3,\ x=-2$

↑ same as zero ∴ hole

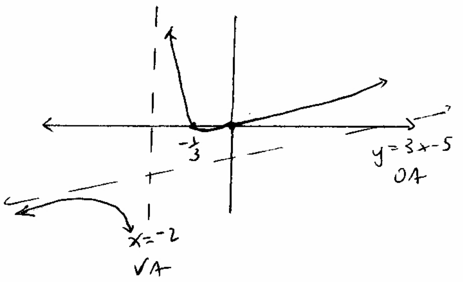
VA

hole: $y=\frac{x(3x+1)}{x+2}$

$y=\frac{-3(-8)}{-1}$

$y=-24$

$(-3,-24)$



11) A polynomial of degree 5 has a negative leading coefficient.

a) How many turning points could the polynomial have?
b) How many zeros could the function have?
c) Describe the end behaviour.
d) Sketch two possible graphs, each passing through the point $(1,\ -2)$.

poly of deg 5

$\Rightarrow n=5$

a) max # of turning pts $=n-1$ $=4$

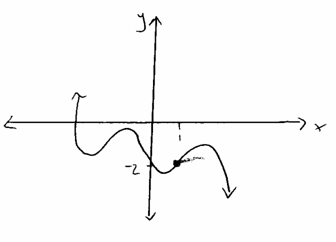
∴ 0, 2 or 4 turning pts

b) max # of zeros $=n$ $=5$ odd deg.

zeros 1, 2, 3, 4 or 5

c) –ve lead. coeff ∴ as $x\to\infty$, $y\to-\infty$

odd deg as $x\to-\infty$, $y\to\infty$



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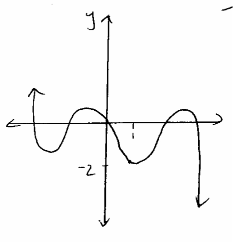
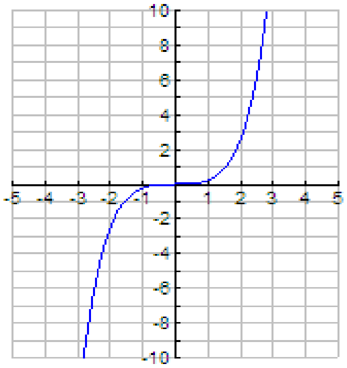


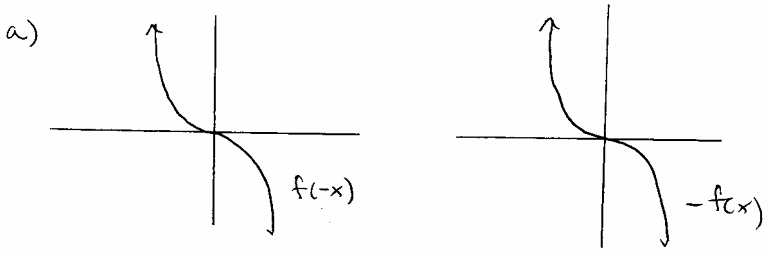
many answers.

Symmetry:

12) Determine whether each of the following functions is even, odd, or neither. Justify your answer.

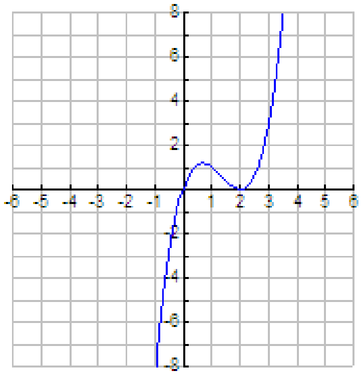
a)

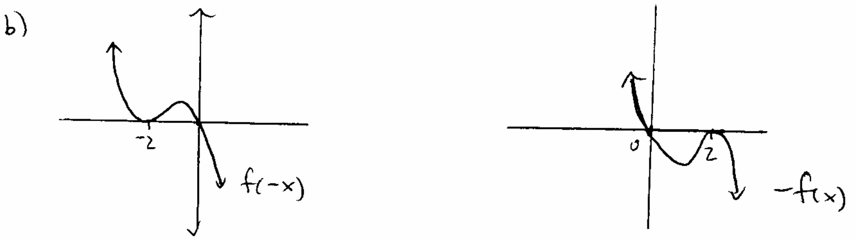




$f(-x) = -f(x)$ ∴ odd fnct.

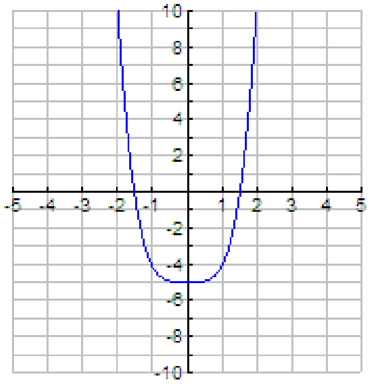
b)

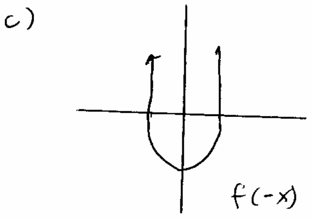




$f(x) \neq f(-x)$, $f(-x) \neq -f(x)$ ∴ neither odd nor even

c)





$f(-x) = f(x)$ ∴ fnct is even

d) $f(x) = 3x^2 + 4$

d) $f(x) = 3x^2 + 4$

$f(-x) = 3(-x)^2 + 4$

$f(-x) = 3x^2 + 4$

$f(-x) = f(x)$

∴ fnct is even

e) $f(x) = -3x^3 + x$

e) $f(x) = -3x^3 + x$

$f(-x) = -3(-x)^3 + (-x)$

$f(-x) = 3x^3 - x$

$-f(x) = -(-3x^3 + x)$

$-f(x) = 3x^3 - x$

$-f(x) = f(-x)$

∴ odd fnct

f) $f(x) = \tan x$

f) $f(x) = \tan x$, assume $x \in QI$

$f(-x) = \tan(-x)$ $-x \in QIV$
tan ratio is $-$ve in QIV

$f(-x) = -\tan x$

$-f(x) = -\tan x$

$-f(x) = f(-x)$ ∴ odd fnct

g) $y = 3^x + 1$

g) $y = 3^x + 1$
let $y = f(x)$
$f(x) = 3^x + 1$
$f(-x) = 3^{-x} + 1$
$-f(x) = -(3^x + 1)$
$-f(x) = -3^x - 1$

$f(-x) \neq f(x)$
$f(-x) \neq -f(x)$
∴ fnct is neither odd nor even.

h) $f(x) = 3\log x - 1$

h) $f(x) = 3\log x - 1$
$f(-x) = 3\log(-x) - 1$
$-f(x) = -(3\log x - 1)$
$-f(x) = -3\log x + 1$
$f(-x) \neq f(x)$
$f(-x) \neq -f(x)$
∴ fnct neither odd nor even

i) $y = \dfrac{1}{x^2 - 4}$

i) $y = \dfrac{1}{x^2-4}$
let $y = f(x)$.
$f(-x) = \dfrac{1}{(-x)^2-4}$
$f(-x) = \dfrac{1}{x^2-4}$
$f(-x) = f(x)$
∴ fnct is even

j) $f(x) = 2^x + 2^{-x}$

j) $f(x) = 2^x + 2^{-x}$
$f(-x) = 2^{-x} + 2^{-(-x)}$
$f(-x) = 2^{-x} + 2^x$
∵ addition is commutative
$f(-x) = 2^x + 2^{-x}$
$f(-x) = f(x)$
∴ fnct is even

k) $f(x) = \dfrac{\sin x}{x^2 - 4}$
(use combinations of functions to justify)

k) $f(x) = \dfrac{\sin x}{x^2-4}$
$y = \sin x$ is an odd fnct
$y = x^2 - 4$ is an even fnct.
The quotient of an odd and an even fnct is odd

l) $f(x) = x \cdot \log x$
(use combinations of functions to justify)

l) $f(x) = x \cdot \log x$
$y = x$ is an odd fnct
$y = \log x$ is neither odd nor even
The product of an odd and a neither is a neither.

Rates of Change:

13) The position in kilometres of a particle at t hours is given by $d(t) = t^3 - 12t^2 + 34t + 75$, where $t \geq 0$.

a) What is the initial position of the particle?
b) What is the particle's average velocity from 3 hours to 5 hours?
c) What is the particle's instantaneous velocity at 7 hours?

a) initial position ⇒ $t = 0$

$d(0) = 75$

The initial position is 75 Km.

b) avg. vel'y = avg. RoC of $d(t)$

$$\text{avg. vel'y} = \frac{d(5) - d(3)}{5 - 3}$$

$$\text{avg. vel'y} = \frac{70 - 96}{2}$$

$$\text{avg. vel'y} = -13$$

The ptcles avg. vel'y from 3 hrs. to 5 hrs. is −13 Km/hr.

or The position is decr. at an avg. RoC of 13 Km/hr.

c)

nbhd pt	pt.	Δt	$\Delta d(t)$	$\frac{\Delta d(t)}{\Delta t}$
(6.9, 66.789)	(7, 68)	0.1	1.211	12.11
(6.99, 67.8709)	(7, 68)	0.01	0.1291	12.91
(7.01, 68.1309)	(7, 68)	−0.01	−0.1309	13.09
(7.1, 69.391)	(7, 68)	−0.1	−1.391	13.91

↓ 13 ↑

The inst. rate of change, at $t = 7$ hrs, is 13 Km/hr.

14) Find the slope of the secant of $y = 2^x - 3$ that passes through the points where $x = -3$ and $x = 1$.

$$y = 2^x - 3$$

$$m_{sec} = \frac{\Delta y}{\Delta x}$$

$$= \frac{y|_{x=1} - y|_{x=-3}}{1 - (-3)}$$

$$\doteq \frac{-1 + 2.875}{4}$$

$$\doteq 0.47$$

15) The concentration of medicine in a patient's bloodstream is given by $C(t)=\dfrac{0.4t}{(0.3t+2)^3}$, $t \geq 0$, where C is measured in milligrams per cubic centimetre and t is the time in hours after the medicine was taken. Determine:

a) the concentration in the bloodstream 3 hours after the medicine was taken.

b) the average rate at which the concentration is decreasing from 4 hours after taking the medicine to 7 hours after taking the medicine.

c) the instantaneous rate of change for the concentration 2 hours after the medicine was taken. Interpret the meaning of your answer.

a) $C(3) = \dfrac{0.4(3)}{[(0.3)(3)+2]^3}$

$C(3) \doteq 0.0492$

After 3 hrs. The conc. is 0.0492 mg/cm³

b) $\text{avg. RoC} = \dfrac{\Delta C(t)}{\Delta t}$

$\text{avg. RoC} = \dfrac{C(7) - C(4)}{7-4}$

$\text{avg. RoC} \doteq \dfrac{0.0406 - 0.0488}{3}$

$\text{avg. RoC} \doteq -0.0027$

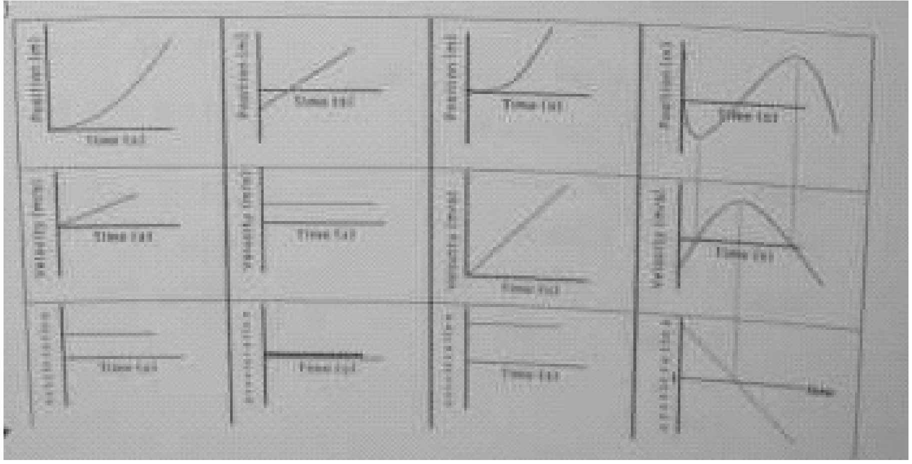
The conc. is decr. at an avg. RoC of 0.0027 mg/cm³/hr.

c)

nbhd Pt	Pt.	Δt	$\Delta C(t)$	$\frac{\Delta C(t)}{\Delta t}$
(1.9, 0.0448)	(2, 0.0455)	0.1	0.0007	0.007
(1.99, 0.04545)	(2, 0.04552)	0.01	0.00007	0.007
(2.01, 0.04559)	(2, 0.04552)	-0.01	-0.00007	0.007
(2.1, 0.0462)	(2, 0.0455)	-0.1	-0.0007	0.007

The inst. RoC is approx. 0.007 mg/cm³/hr. The conc. is increasing slightly.

16) Complete the table



Solving Equations/inequalities:

17) Determine the solution(s) of:

a) $349 = 7(1.49)^x$ b) $\log_2 8 = 3\log_2 x - \log_2 3$

) a) $\frac{349}{7} = \frac{7(1.49)^x}{7}$

$49.8571 \doteq 1.49^x$

$\Leftrightarrow \log_{1.49} 49.8571 \doteq x$

$\frac{\log 49.8571}{\log 1.49} \doteq x$

$9.80 \doteq x$

b) $\log_2 8 = 3\log_2 x - \log_2 3$

$\log_2 8 = \log_2 x^3 - \log_2 3$

$\log_2 8 = \log_2 \left(\frac{x^3}{3}\right)$

$\therefore \quad 8 = \frac{x^3}{3}$

$24 = x^3$

$\sqrt[3]{24} = x$

$2.88 \doteq x, \quad x > 0$

c) $7^x = 3^{x^2-1}$

$\Leftrightarrow \log_3 7^x = x^2 - 1$

$x(\log_3 7) = x^2 - 1$

$1.7712x \doteq x^2 - 1$

$0 \doteq x^2 - 1.7712x - 1$

$x \doteq \frac{1.7712 \pm \sqrt{(-1.7712)^2 - 4(-1)}}{2}$

$x \doteq 2.22$ or $x \doteq -0.45$

$\longrightarrow$ $\underline{\underline{\text{or}}}$

$\therefore \log 7^x = \log 3^{x^2-1}$

$x \log 7 = (x^2-1)\log 3$

$0.8451x \doteq 0.4771x^2 - 0.4771$

$0 \doteq 0.4771x^2 - 0.8451x - 0.4771$

$x \doteq \frac{0.8451 \pm \sqrt{(-0.8451)^2 - 4(-0.4771)^2}}{2(0.4771)}$

$x \doteq 2.22$ or $x \doteq -0.45$

d) $x^3 - 6x^2 + 5x + 12 > 0$

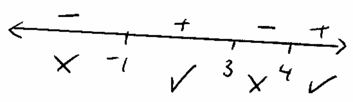
$x^3 - 6x^2 + 5x + 12 > 0$

let $P(x) = x^3 - 6x^2 + 5x + 12$

$P(-1) = 0 \therefore x+1$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 5 & 12 \\ \hline & \downarrow & -1 & 7 & -12 \\ & 1 & -7 & 12 & 0 \end{array}$$

$\therefore (x+1)(x^2 - 7x + 12) > 0$



when $-1 < x < 3$ and

when $4 < x$

or use factor chart

factors	$x < -1$	$-1 < x < 3$	$3 < x < 4$	$4 < x$
$x+1$	−	+	+	+
$x-4$	−	−	−	+
$x-3$	−	−	+	+
	−	+	−	+

or sketch!

e) $\frac{4x+9}{4x-1} \le \frac{x+5}{x}$

$$\frac{4x+9}{4x-1} - \frac{x+5}{x} \le 0$$

$$\frac{x(4x+9)-(4x-1)(x+5)}{x(4x-1)} \le 0$$

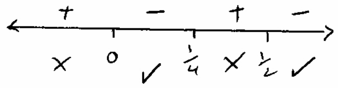
$$\frac{4x^2+9x-4x^2-19x+5}{x(4x-1)} \le 0$$

$$\frac{-10x+5}{x(4x-1)} \le 0$$

$$\frac{-5(2x-1)}{x(4x-1)} \le 0$$

zeros: $\frac{1}{2}$

VA: $x=0$ $x=\frac{1}{4}$



when $0 < x < \frac{1}{4}$

or $\frac{1}{2} \le x$

or

factors	$x<0$	$0<x<\frac{1}{4}$	$\frac{1}{4}<x<\frac{1}{2}$	$\frac{1}{2}<x$
-5	−	−	−	−
$2x-1$	−	−	−	+
x	−	+	+	+
$4x-1$	−	−	+	+
sign of fnct	+	⊖	+	⊖

f) $\cos 2\theta = -0.9541$

zu

$\cos 2\theta = -0.9541$

Let $x = 2\theta$

$\cos x = -0.9541$

$\cos x_r = 0.9541$

$x_r \doteq 0.3042$

period $= \frac{2\pi}{2}$

$= \pi$

$x_1 = \pi - x_r$

$x_1 \doteq 2.8374 \longrightarrow \theta_1 = x_1 \div 2$ $\qquad \theta_3 = \theta_1 + \text{period}$

$\theta_1 \doteq 1.4187$ $\qquad \theta_3 \doteq 4.5603$

$x_2 = \pi + x_r$

$x_2 \doteq 3.4458 \longrightarrow \theta_2 = x_2 \div 2$ $\qquad \theta_4 = \theta_2 + \text{period}$

$\theta_2 \doteq 1.7229$ $\qquad \theta_4 \doteq 4.8645$

for more answers add/subtract $n\pi$

$n \in \mathbb{Z}$.

g) $\sin\theta - \sin\theta \tan\theta = 0$

$\sin\theta - \sin\theta \tan\theta = 0$

$\sin\theta \, (1 - \tan\theta) = 0$

$\sin\theta = 0$ or $1 - \tan\theta = 0$

$\theta = 0, \pi, 2\pi$

$1 = \tan\theta$

$\frac{\pi}{4} = \theta_r$

$\theta_s = \frac{\pi}{4} + \pi$

$\theta_s = \frac{5\pi}{4}$

$\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

h) $6\sin^2\theta - 5\cos\theta - 2 = 0$

) $6\sin^2\theta - 5\cos\theta - 2 = 0$

$6(1 - \cos^2\theta) - 5\cos\theta - 2 = 0$

$6 - 6\cos^2\theta - 5\cos\theta - 2 = 0$

$-6\cos^2\theta - 5\cos\theta + 4 = 0$

$6\cos^2\theta + 5\cos\theta - 4 = 0$

$P = -24$

$S = 5$

$\frac{8}{6}, \frac{-3}{6}$

↓

$\frac{4}{3}, \frac{-1}{2}$

$(2\cos\theta - 1)(3\cos\theta + 4) = 0$

$\cos\theta = \frac{1}{2}$ $\quad \cos\theta = \frac{-4}{3}$

$\theta_1 = \frac{\pi}{3}$ $\quad \cos\theta_R = \frac{+4}{3}$

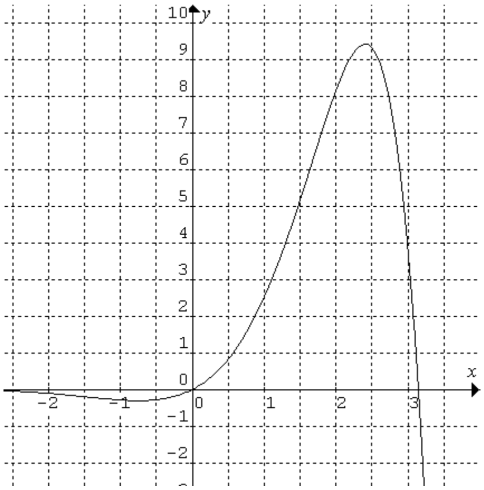
$\theta_2 = 2\pi - \theta_1$ $\quad$ no sol^n $\frac{4}{3} > 1$

$\theta_2 = \frac{5\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

18) The graph of the function $p(x) = 3^x \sin x$ is shown on the right. Use the graph to estimate the answer to the following questions then verify your answer(s) using the equation.

a) evaluate $p(1)$



a) $p(1)$ find y value when $x = 1$

$p(1) \doteq 2.6$

verify: $p(1) = 3^1 \sin 1$

$p(1) \doteq 3(0.8414\ldots)$

$p(1) \doteq 2.52$

b) solve for a if $p(a) = 8$

$p(a) = 8$ find x when $y = 8$

$a \doteq 1.99$ $a \doteq 2.7$

c) the interval for which $y \le 2$

$y \le 2$

find when $y = 2$

$x \doteq 0.85$ $x \doteq 3.07$

verify $p(0.85) \doteq 1.91$ $p(3.07) \doteq 2.09$

$\underline{ok}$ $\underline{ok}$

$y \le 2$ when $x < 0.85$

or $x > 3.07$

19) For the function defined by $f(x) = k(x+1)^2(x-2)(x-4)$

a) Determine the value of k, if (1, -24) is a point on the graph of the function

a) $-24 = K(2)^2(-1)(-3)$

$-24 = 12K$

$-2 = K$

$f(x) = -2(x+1)^2(x-2)(x-4)$

b) solve for p if (3, p) is a point on the graph of the function

$p = f(3)$

$p = -2(4)^2(1)(-1)$

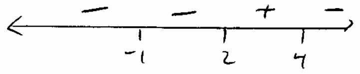
$p = 32$

c) considering the end behaviours and the zeros, state where $f(x) > 0$

c) zeros: -1 DR $\therefore$ sign does not change

2, 4 SR $\therefore$ sign changes

lead coeff. $-$ve $\therefore$ ends $-$ve



$f(x) > 0$ when $2 < x < 4$

Combination of functions:

20) Determine $h(x) = (f \circ g)(x)$ when $f(x) = 2x^4 - 3x^2$ and $g(x) = \sqrt{x-3}$ and state the domain and range of *h(x)*.

a) $D_h \in D_g$ when $g(x) \in D_f$

$D_g: x \geq 3$ and all $g(x) \in D_f$

$D_h: \{x \mid x \geq 3, \ x \in \mathbb{R}\}$

b) $R_h \in R_f$ when $g(x) \in D_f$

$R_f: y \geq -1.125$ and all $g(x) \in D_f$

$R_h: \{y \mid y \geq -1.125, \ y \in \mathbb{R}\}$

↑ min. value of f(x)

21) Given $D_f = \left\{x \middle| \ -5 \leq x \leq 8, \ x \in \Re \right\}$ and $D_g = \left\{x \middle| \ -12 \leq x \leq 3, \ x \in \Re \right\}$ determine

a) D_{f+g} b) $D_{g \cdot f}$

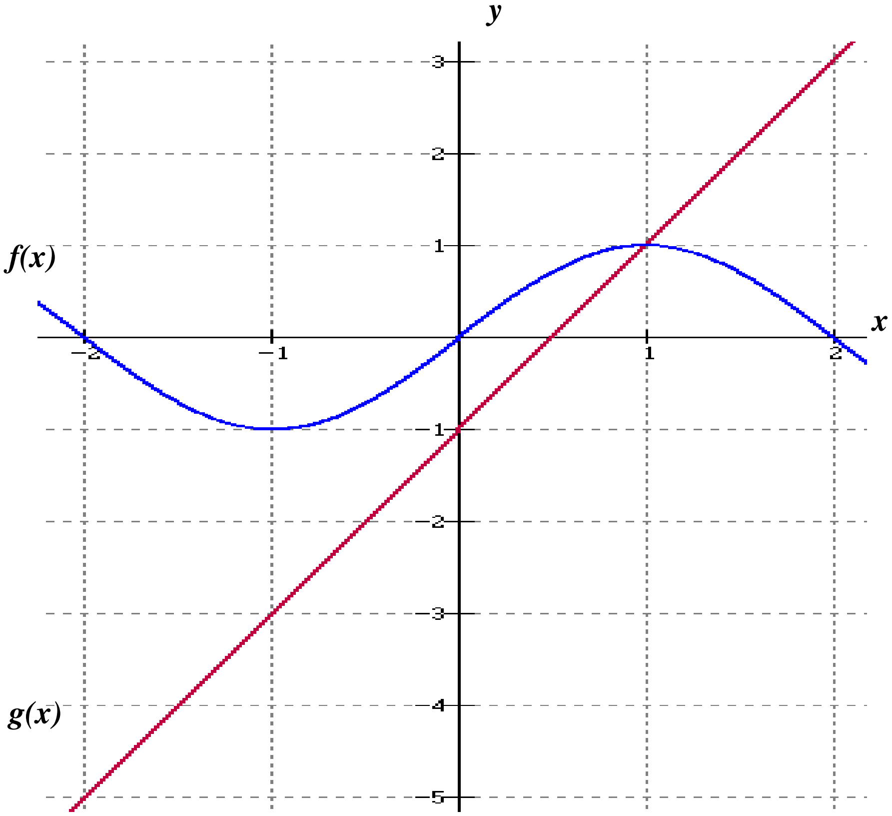
a) D_{f+g} : "overlap"

$D_{f+g} = \{x \mid -5 \leq x \leq 3, \ x \in \mathbb{R}\}$

b) $D_{g \cdot f}$: "overlap"

$D_{g \cdot f} = \{x \mid -5 \leq x \leq 3, \ x \in \mathbb{R}\}$

22) The Graphs of $y = f(x)$ and $y = g(x)$ are given below.



On the same grid sketch a) $y = f(x) + g(x)$ b) $y = f(x) \cdot g(x)$

23) Given $s(x) = \sin x + 2\cos x$,

a) determine D_s b) at most, what is the range of the function?

a) $D_s = \{x \mid x \in \mathbb{R}\}$ (both fnct have the same domain, $x \in \mathbb{R}$)

b) at most $R_s = R_{\sin x} + R_{2\cos x}$

but since phase shift, never equal to the actual sum!

$R_s : \{y \mid -3 < y < 3, y \in \mathbb{R}\}$

24) Given $d(x) = \tan x + \log x$, determine D_d

$D_d : \{x \mid x > 0, \quad x \neq \frac{\pi}{2} + n\pi, n \in N, x \in \mathbb{R}\}$

↑ for log x

↑ restrictions on tan x

25) What is the maximum number of zeros possible for $p(x) = (x+2)(x+3)(x-4)\log x$? Do you think there will actually be that many zeros? Justify your answers.

$p(x) = (x+2)(x+3)(x-4)\log x$

$p(x) =$ cubic · logarithm

The fnct could have 4 zeros (3 from cubic an 1 from log)

This will not be the actual # of zeros since two of the zeros of the cubic are negative and the domain – because of $\log x$ – is restricted to $x > 0$.

actual zeros = 1, 4

26) Given $f(x) = \dfrac{x}{x+1}$ and $g(x) = \cos x$, determine (Keep your answers within $[0, 2\pi]$.)

a) $D_{f \cdot g}$ b) $D_{f \div g}$ c) zeros, holes and vertical asymptotes of $g \div f$

a) $D_{f \cdot g}$: overlap of D_f and D_g

$D_{f \cdot g} = \{x \mid 0 \le x \le 2\pi, x \in \mathbb{R}\}$

$x \neq -1$ not needed ∵ not in domain

b) $D_{f \div g}$: overlap of D_f and D_g additional restrictions $g(x) \neq 0$

$D_{f \div g} = \{x \mid x \neq \frac{\pi}{2}, \frac{3\pi}{2}, 0 \le x \le 2\pi, x \in \mathbb{R}\}$

restriction $x \neq -1$ not needed ∵ not in D_g

c) zeros: $\frac{\pi}{2}, \frac{3\pi}{2}$; hole: none since x = -1 is not in the domain; VA x = 0

Word Problems:

27) Distance in kilometres above sea level is given by the formula $d = \dfrac{500(\log P - 2)}{27}$, where P is the atmospheric pressure measured in kiloPascals, kPa.

a) At the top of the highest mountain in Shelbyville, the atmospheric pressure was recorded as being 220 kPa. Calculate the height of the mountain above sea level.

$P = 220$

$d = \frac{500(\log 220 - 2)}{27}$

$d \doteq 6.34$

The highest mountain in Shelbyville is approx. 6.34 km above sea level.

b) The town of Springfield has a mountain with a peak 4.5 km above sea level. Calculate the atmospheric pressure at the top of the mountain.

$4.5 = \frac{500(\log P - 2)}{27}$

$121.5 = 500(\log P - 2)$

$0.243 = \log P - 2$

$2.243 = \log P$

$\Leftrightarrow \quad P = 10^{2.243}$

$P \doteq 174.98$

The atmospheric pressure at the top of the mountain is approx. 175 kPa.

c) In the year 1980, both towns had an earthquake. Springfield's earthquake measured 7.5 on the Richter Scale. The magnitude of the earthquake was $10^{7.5}$. The earthquake in Shelbyville measured 6.4. How many times more intense was Springfield's earthquake when compared to Shelbyville's earthquake. Recall: $M = \log\left(\frac{I}{I_o}\right)$

c) $M = \log \frac{I}{I_o}$

$\Delta M = \log \frac{I_{springfield}}{I_{shelbyville}}$

$7.5 - 6.4 = \log \frac{I_{spring}}{I_{shelby}}$

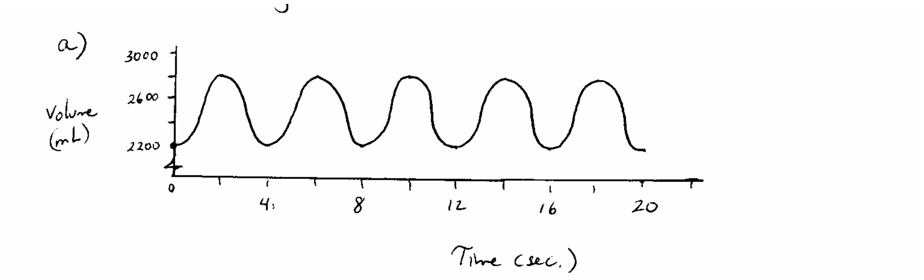
$\Leftrightarrow \quad \frac{I_{spring}}{I_{shelby}} = 10^{1.1}$

$\frac{I_{spring}}{I_{shelby}} \doteq 12.59$

Springfield's earthquake is approx. 12.59 times more intense than Shelbyville's earthquake.

28) The volume of air in the lungs during normal breathing can be modeled by a sinusoidal function of time. Suppose a person's lungs contain from 2200 mL to 2800 mL of air during normal breathing. Suppose a normal breath takes 4 seconds, and that $t = 0\ s$ corresponds to a minimum volume.

a) Let *V* represent the volume of air in a person's lungs. Draw a graph of Volume versus time for 20 seconds.



b) State the period, amplitude, phase shift and vertical translation for the function.

Period = 4

$K = \frac{2\pi}{4}$

$K = \frac{\pi}{2}$

Amplitude = $\frac{|Max - Min|}{2}$

= 300

P.S: 0 for − cosine

V. translation = $\frac{Max + Min}{2}$

= 2500

c) Write a possible equation for the volume of air as a function of time.

$V(t) = -300\cos\frac{\pi}{2}t + 2500$

where t rep. the time in seconds and V(t) is the volume in mL.

d) Describe how the graph would change if the person breaths more rapidly.

If a person breaths more rapidly the period will be shorter ∴ K would be larger.

e) Describe how the graph would change if the person took bigger breaths.

If a person took deeper breaths the amplitude would be greater since they would breath in more air.
(The minimum amount would probaby stay the same ∴ graph would also shift up).

f) Determine the amount of air in the lungs after 8 seconds.

$V(8) = -300\cos\frac{\pi}{2}\cdot 8 + 2500$

$V(8) = -300\cos 4\pi + 2500$

$V(8) = 2200$ (matches graph)

After 8 seconds the is 2200mL of air in the lungs.

g) Determine when, within the first 8 seconds, the volume is 2400 mL.

$V(t) = 2400$

$2400 = -300 \cos \frac{\pi}{2}t + 2500$

$2400 - 2500 = -300 \cos \frac{\pi}{2}t$

$-100 = -300 \cos \frac{\pi}{2}t$

$\frac{1}{3} = \cos \frac{\pi}{2}t$

Let $\theta = \frac{\pi}{2}t$

$\frac{1}{3} = \cos \theta$

$1.2310 \doteq \theta_1 \rightarrow t_1 = \theta_1 \div \frac{\pi}{2}$

$t_1 = 0.7837$

$\theta_2 = 2\pi - \theta_1$

$\theta_2 \doteq 5.0522 \rightarrow t_2 \doteq 3.2163$

$t_3 = t_1 + \text{period}$

$t_3 = 4.7837$

$t_4 = t_2 + \text{period}$

$t_4 = 7.2163$

$t_5 = t_3 + \text{period}$

> 8 second ∴ inadmissible

The volume is 2400 mL when $t \doteq 0.78$ sec, 3.22 sec, 4.78 sec, and 7.22 sec.

29) You have just walked out the front door of your home. You notice that it closes quickly at first and then closes more slowly. In fact, a model of the movement of a closing door is given by $d(t) = 200t(2)^{-t}$, where *d* represents the width of the opening in cm *t* seconds after opening the door.

a) determine the width of the opening after 2 sec., 4 sec., 6 sec, 10 sec.

a) $d(2) = 200(2)(2)^{-2}$

$d(2) = 100$

$d(4) = 200(4)(2)^{-4}$

$d(4) = 50$

$d(6) = 200(6)(2)^{-6}$

$d(6) = 18.75$

$d(10) = 200(10)(2)^{-10}$

$d(10) \doteq 1.95$

The door opening is 100 cm after 2sec, 50cm after 4sec, 18.75cm after 6 sec and 1.95 cm after 10 sec.

b) Determine the average rate of change from t = 0 sec. to t = 1.5 sec.. What does this tell you about the movement of the door.

$\text{avg. RoC} = \frac{d(1.5) - d(0)}{1.5 - 0}$

$\text{avg. RoC} \doteq \frac{106.07 - 0}{1.5}$

$\text{avg. RoC} \doteq 70.71$

The door is opening at a rate of 70.71 cm/sec.

c) Determine the average rate of change from t = 3 sec. to t = 6 sec.. What does this tell you about the movement of the door.

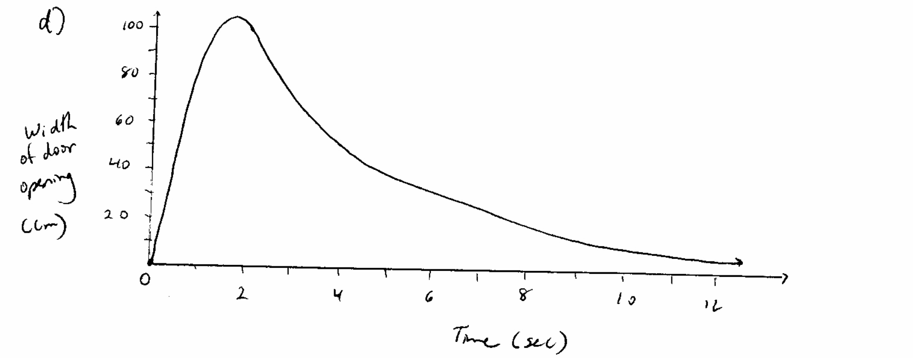
$\text{avg. RoC} = \frac{d(6) - d(3)}{6 - 3}$

$\text{avg. RoC} = \frac{18.75 - 75}{3}$

$\text{avg. RoC} = -18.75$

The door is closing at a rate of 18.75 cm/sec.

d) sketch a graph to model the movement of the door. How does your sketch support the conclusions you reached in b) and c)?



Identities:

30) Prove the following identities.

a) $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}=\dfrac{2}{\cos\theta}$ b) $\dfrac{1}{\sec\theta+\tan\theta}=\sec\theta-\tan\theta$

a) $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}=\dfrac{2}{\cos\theta}$

LS: $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}$

$=\dfrac{\cos\theta(1-\sin\theta)+\cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$=\dfrac{\cos\theta-\sin\theta\cos\theta+\cos\theta+\sin\theta\cos\theta}{1-\sin^2\theta}$

$=\dfrac{2\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta}$

$=\dfrac{2}{\cos\theta}$

$=RS$ QED

LS: $\dfrac{1}{\sec\theta+\tan\theta}$

$=1\div(\sec\theta+\tan\theta)$

$=1\div\left(\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}\right)$

$=1\div\dfrac{1+\sin\theta}{\cos\theta}$

$=1\cdot\dfrac{\cos\theta}{1+\sin\theta}$

$=\dfrac{\cos\theta}{1+\sin\theta}\cdot\dfrac{1-\sin\theta}{1-\sin\theta}$

$=\dfrac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$=\dfrac{\cancel{\cos\theta}(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$

$=\dfrac{1-\sin\theta}{\cos\theta}$

$=\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$

$=\sec\theta-\tan\theta$

$=RS$ QED

c) $\frac{1}{1+\sin\theta} = \sec^2\theta - \frac{\tan\theta}{\cos\theta}$

RS: $\sec^2\theta - \frac{\tan\theta}{\cos\theta}$

$= \frac{1}{\cos^2\theta} - \tan\theta \cdot \frac{1}{\cos\theta}$

$= \frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos^2\theta}$

$= \frac{1-\sin\theta}{\cos^2\theta}$

$= \frac{1-\sin\theta}{1-\sin^2\theta}$

$= \frac{\cancel{1-\sin\theta}}{(1+\sin\theta)\cancel{(1-\sin\theta)}}$

$= \frac{1}{1+\sin\theta}$

$=$ LS QED

d) $(1-\cos\beta)^2 + \sin^2\beta = 2(1-\cos\beta)$

LS: $(1-\cos\beta)^2 + \sin^2\beta$

$= 1 - 2\cos\beta + \cos^2\beta + \sin^2\beta$

$= 1 - 2\cos\beta + 1$

$= 2 - 2\cos\beta$

$= 2(1-\cos\beta)$

$=$ RS QED

Miscellaneous:

31) $2x^3 + 3x^2 + kx - 5$ is divided by $x+2$ to give a remainder of 2. Determine *k*.

let $P(x) = 2x^3 + 3x^2 + Kx - 5$

remainder thm: $P(-2) = 2$

$2(-2)^3 + 3(-2)^2 + K(-2) - 5 = 2$

$-16 + 12 - 2K - 5 = 2$

$-2K = 11$

$K = \frac{-11}{2}$

32) State the quotient and remainder when $2x^3 + 5x^2 - x - 5$ is divided by $x + 2$.

$x+2 \,\big)\, 2x^3 + 5x^2 - x - 5$

$-(2x^3 + 4x^2)$

$x^2 - x$

$-(x^2 + 2x)$

$-3x - 5$

$-(-3x - 6)$

1

quotient: $2x^2 + x - 3$

remainder: 1

33) Use the Factor Theorem to fully factor: $x^3 - 4x^2 - 11x + 30$

Let $P(x) = x^3 - 4x^2 - 11x + 30$

$P(2) = 8 - 16 - 22 + 30$

$P(2) = 0 \therefore x-2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & \downarrow & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$x^3 - 4x^2 - 11x + 30 = (x-2)(x^2 - 2x - 15)$

$x^3 - 4x^2 - 11x + 30 = (x-2)(x-3)(x+5)$

34) Convert the following radians to degrees. Round your answer to one decimal place, if necessary.

a) $\dfrac{5\pi}{6}$ b) $\dfrac{-3\pi}{8}$ c) 2.678

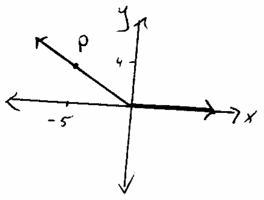
34) a) $\frac{5\pi}{6} = \frac{5}{6}\pi \cdot \frac{180}{\pi}$ b) $\frac{-3\pi}{8} = \frac{-3}{8}\pi \cdot \frac{180}{\pi}$ $2.678 = 2.678 \cdot \frac{180}{\pi}$

$= 150°$ $= -67.5°$ $\doteq 153.4°$

35) The point P(-5, 4) is on the terminal arm of an angle of measure θ in standard position.

a) Sketch the principal angle. b) Determine the exact value of $\sin\theta$.

a)



b) $x^2 + y^2 = r^2$

$25 + 16 = r^2$

$41 = r^2$

$\sin\theta = \frac{4}{\sqrt{41}}$

c) Determine the exact value of $\cos\left(\theta - \dfrac{\pi}{6}\right)$

c) $\cos(\theta - \frac{\pi}{6})$

$= \cos\theta \cos\frac{\pi}{6} + \sin\theta \sin\frac{\pi}{6}$

$= \frac{-5}{\sqrt{41}} \cdot \frac{\sqrt{3}}{2} + \frac{4}{\sqrt{41}} \cdot \frac{1}{2}$

$= \frac{-5\sqrt{3} + 4}{2\sqrt{41}}$

d) Determine the value of θ, to the nearest degree, where $0 \le \theta \le 2\pi$.

$\theta = \sin^{-1}\left(\frac{4}{\sqrt{41}}\right)$

$\theta \doteq 0.6747$

36) Determine the smallest positive co-terminal angle to $\frac{13\pi}{5}$. Determine a co-terminal angle that is larger than $\frac{13\pi}{5}$. Determine a negative co-terminal angle.

+ve co-terminal $= \frac{13\pi}{5} - 2\pi$

$= \frac{3\pi}{5}$

−ve coterminal

$= \frac{3\pi}{5} - 2\pi$

$= -\frac{7\pi}{5}$

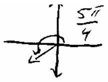
larger co-terminal $= \frac{13\pi}{5} + 2\pi$

$= \frac{23\pi}{5}$

37) Determine the exact value for each of the following:

a) $\sin\frac{5\pi}{4}$ b) $\cos\frac{11\pi}{6}$

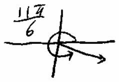
a) $\sin\frac{5\pi}{4}$



$= -\sin\frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

b) $\cos\frac{11\pi}{6}$



$= \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

c) $\tan\frac{\pi}{8}$

$\tan\frac{\pi}{4} = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$1 = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$1-\tan^2\frac{\pi}{8} = 2\tan\frac{\pi}{8}$

$0 = \tan^2\frac{\pi}{8} + 2\tan\frac{\pi}{8} - 1$

$\tan\frac{\pi}{8} = \frac{-2 \pm \sqrt{4+4}}{2}$

$\tan\frac{\pi}{8} = \frac{-2 \pm \sqrt{8}}{2}$

$\tan\frac{\pi}{8} = \frac{-2 \pm 2\sqrt{2}}{2}$

$\tan\frac{\pi}{8} = \frac{-2 \pm \sqrt{2}}{2}$

d) $\csc\frac{7\pi}{12}$

d) $\csc \frac{7\pi}{12}$ = reciprocal of $\sin \frac{7\pi}{12}$

$\sin \frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$\sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$\sin \frac{7\pi}{12} = \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{3} + \cos\frac{\pi}{4} \cdot \sin\frac{\pi}{3}$

$\sin \frac{7\pi}{12} = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$\sin \frac{7\pi}{12} = \frac{\sqrt{2} + \sqrt{6}}{4}$

$\csc \frac{7\pi}{12} = \frac{4}{\sqrt{2}+\sqrt{6}}$

38) Express each of the following as a co-function

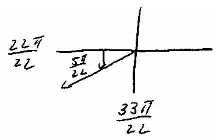
a) $\sin\dfrac{27\pi}{22}$

b) $\sec\dfrac{17\pi}{30}$

a) $\sin \frac{27\pi}{22}$

$= \cos \frac{6\pi}{22}$

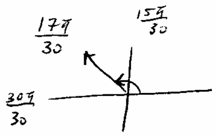
$= \cos \frac{3\pi}{11}$



b) $\sec \frac{17\pi}{30}$

$= -\csc \frac{2\pi}{30}$

$= -\csc \frac{\pi}{15}$



39) Express $\log_{11} 3 + 2\log_{11} 5 - \log_{11} 7$ as a single logarithm.

$\log_{11} 3 + 2\log_{11} 5 - \log_{11} 7$

$= \log_{11} 3 + \log_{11} 25 - \log_{11} 7$

$= \log_{11} \frac{75}{7}$

40) Express $\log_4 7$ as a single logarithm with base 2.

$\log_4 7 = \frac{\log_2 7}{\log_2 4}$

$\log_4 7 = \frac{\log_2 7}{2}$

$\log_4 7 = \frac{1}{2} \log_2 7$

$\log_4 7 = \log_2 \sqrt{7}$