21st / Morning

MCR3U

FINAL EXAM REVIEW

As the end of semester two swiftly approaches, preparation for the MCR3U examination is of vital importance. The following is a recommended guideline for studying and preparing for this final exam:

1. FINAL REVIEW HANDOUT: Complete the exam review provided.

2. TESTS and QUIZZES: Study the questions on all previous tests and quizzes.
   Reattempt any questions you answered incorrectly.
   If you cannot answer a question, seek out any help available. (eg. see your teacher, parents, friends, etc.)
   Extra help is Monday to Friday at lunch until exams.

3. GROUP STUDY SESSIONS: It may be helpful to plan group study sessions with your friends. Choose a specific theme (eg. Sinusoidal Functions), and bring quizzes and tests along to discuss/do particular questions.
   HINT: This is most helpful if the group is NOT too large, and remains focused on the areas of concern.

Remember that planning and preparation is the key to success.

DO NOT WAIT UNTIL THE LAST MINUTE TO STUDY.

Everything in the curriculum can reasonably be included on the exam. Exam review sheets highlight some topics to focus on.

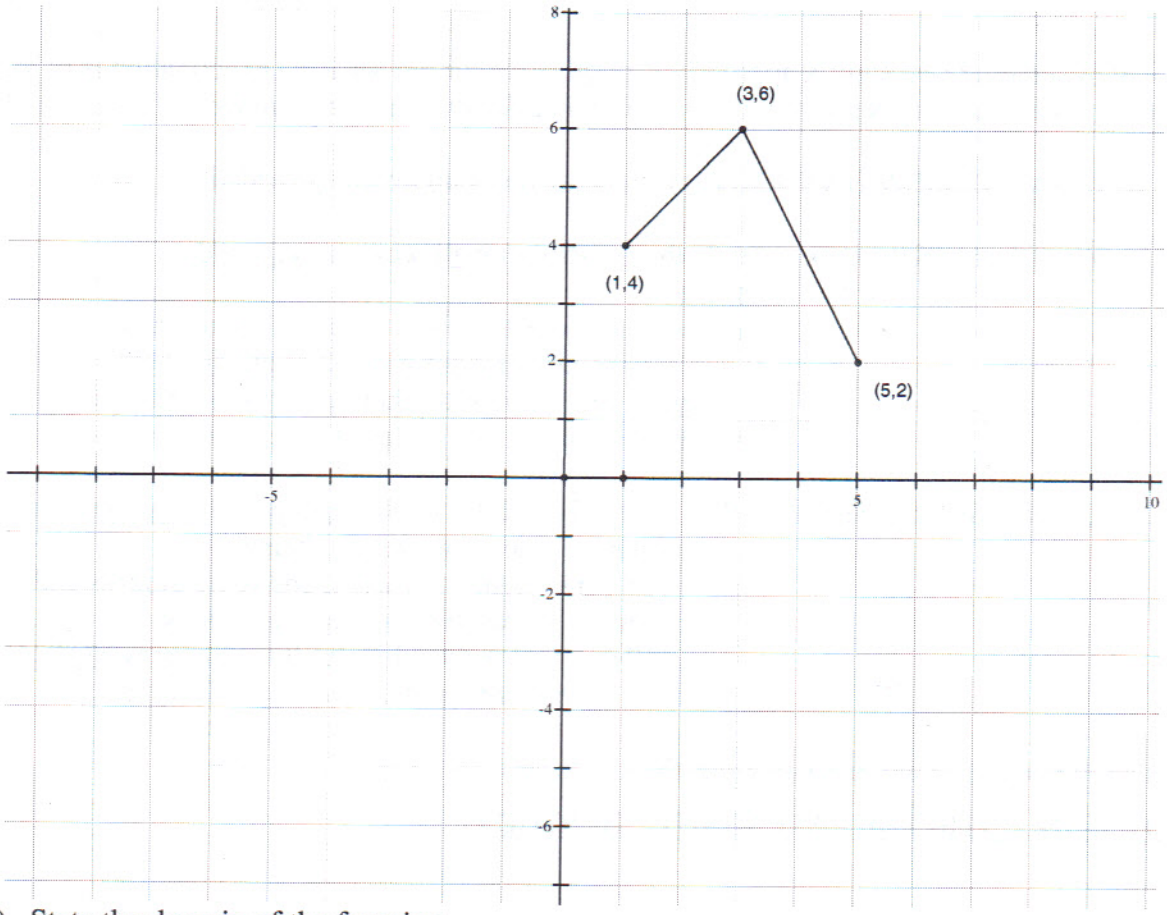
STUDY QUESTIONS

Do: Cumulative Review page 206 #1-32
Do: Cumulative Review page 408 #1-28
Do: Cumulative Review page 538 #2, 9-10

Unit 1 – Functions

1. a) Determine the equation for the inverse of $f(x)=\frac{3}{x}-2$. Express your final answer in function notation.
   b) What is the domain and range of f?
   c) What is the domain and range of f^{-1}?

2. Consider the function f whose graph is shown below



a) State the domain of the function.

b) Determine the value(s) of $f(2)$ _____.

c) Determine the value(s) of $f^{-1}(5)$ _____.

On the same set of axes as the original graph, draw the graph of

d) $y = f(2x)$

e) $y = -f^{-1}(x)$

f) $y = f(x+6)-3$

3. a) Sketch the graph of each of the following:

i) $f(x) = -\sqrt{1-x}$

ii) $g(x) = 4(x-3)^2 + 9$

iii) $h(x) = -\left|\frac{1}{3}x\right| - 2$

iv) $p(x) = \frac{2}{x+1}$

b) State whether or not each of the above is a function and why.

c) State the domain and range of each of the relations.

4. Describe the transformations required to change the graph of $f(x)=\sqrt{x}$ into the graph of $f(x)=\sqrt{2(x+3)}-1$. Make sure that your transformations are in the correct order.

5. Describe the transformations required to change the graph of $f(x)$ into $g(x)$ when $g(x) = -3f(-2x-6)-7$. Make sure that your transformations are in the correct order.

6. Given $g(x)=-x^2+7x+1$, determine $g(3)$.

7. Given $f(x) = 2x - 1$, determine $f^{-1}(7)$.

8. a) Given $f(t) = -3t^2 - 18t + 3$, determine the equation of the inverse of f.
   b) Is f^{-1} a function?
   c) What restriction could you place on f to ensure the inverse is a function?

Unit 2 – Algebra

9. Simplify each of the following:

   a) $4(2x - 2)^2 - (2x - 5)(3x + 3)$

   b) $5[2-(4g+2)] - [3g-4(g-1)]$

   c) $(t-3)^3 - t$

   d) $2m^2[(3m-1)+4] - (9-m)$

10. Simplify each of the following. State any restriction(s) on the variables.

   a) $\dfrac{-9xy^2}{6x} \bullet \dfrac{15x^2}{-12y}$

   b) $\dfrac{3x^2-9x}{5x^2-14x-3}$

   c) $\dfrac{x^2-4}{x+3} \div \dfrac{x+2}{x^2-9}$

   d) $\dfrac{x+6}{x^2+9x+18} - \dfrac{3x-9}{x^2-2x-3}$

   e) $\dfrac{-4(3-p)}{6(p-3)}$

   f) $\dfrac{6k-2}{-2k}$

   g) $\dfrac{6f^2-f-12}{2f^2-3f-9} \bullet \dfrac{2f^2+3f}{16-9f^2}$

   h) $\dfrac{8x}{x+1} + \dfrac{2x}{x+2}$

11. Simplify

   a) $4\sqrt{288}$

   b) $-\sqrt{90}+5\sqrt{40}$

   c) $12\sqrt{27} - 13\sqrt{200} - 3\sqrt{48} + 7\sqrt{8}$

   d) $(-\sqrt{5})(2\sqrt{10})$

12. Simplify

   a) $(5+2\sqrt{32})-(2-3\sqrt{18})$

   b) $(1-2\sqrt{12})(3-8\sqrt{2})$

Unit 3 – Quadratics

13. Solve for x. Leave all answers in exact (simplified radical) form. No decimals.

a) $2x^2 + 4x - 3 = 0$ b) $x^2 - 12 = 0$

14. Given $f(x) = -2x^2 - 5x + 1$,
a) Determine the value of x that will produce the maximum or minimum value of the function.
b) Determine if the function has a maximum or minimum.

15. For what value(s) of m does $mx^2 - 3x + 12 = 0$ have no real roots?

16. Without completing the square or solving for x, determine how **many** real roots the relation has.
$f(x) = 3x^2 - x + 3$

17. The revenue function for a new product is, $R(x) = -2x^2 + 17x + 4$ where R is the revenue in thousands of dollars and x is the number sold in thousands. The cost function is $C(x) = 5x + 8$.

a) Determine the maximum revenue.

b) Determine the corresponding profit function, *in simplest form.*

c) How many items must be sold to maximize profit?

18. a) What is the domain and range of the function $f(x) = -5(x + 7)^2 - 3$?

b) Explain how you can determine the number of roots of the equation without expanding into standard form or solving for x. State the number of roots in your explanation.

19. The demand function for a new product is $p(x) = -2x + 7$, where p is the selling price of the product and x is the number sold in thousands. The cost function is $C(x) = x + 2$.

a) Determine an equation for **the profit**, in standard form and in function notation.
b) How many units must be sold for the company to break even?

20. Solve the following linear-quadratic system algebraically: $2x - 3y + 5 = 0$
$xy = 12$

21. Solve the following linear-quadratic system graphically: $x + y = 8$
$x^2 - 10x + 20 = y$

Unit 4– Exponential Functions

22. Write each term as a radical, then evaluate the following, if possible:

a) $-81^{\frac{-3}{4}}$ b) $\left(\frac{16}{25}\right)^{-\frac{1}{2}}$

c) $32^{\frac{3}{5}} - 27^{\frac{-2}{3}} - 5^0$ d) $\left(\frac{23}{27}\right)^{1.6} + \left(\frac{4}{9}\right)^{-2.5}$

23. Simplify. Your answers should contain as few terms as possible and should contain only positive exponents.

a) $\left(243x^{10}\right)^{\frac{4}{5}}$ b) $\left(\sqrt[4]{3}x^{-2}y^{3}\right)^{12}$ c) $\sqrt{\dfrac{50x^{2}y^{4}}{5x^{4}y^{7}}}$

d) $\dfrac{(m^{2}n)^{-\frac{5}{2}}}{(m^{3}n^{-3})^{\frac{1}{6}}}$ e) $\dfrac{(m^{2n})(m^{-3n})(m^{n})}{m^{n}}$

24. Graph the function $f(x) = 2^{x}$. On the same set of axes, graph the function $f(x) = 3(2)^{x} + 1$

25. State the transformations applied to the graph of $g(x) = 2^{x}$ to arrive at $f(x) = -\frac{1}{2}(2)^{-3x-6} + 1$

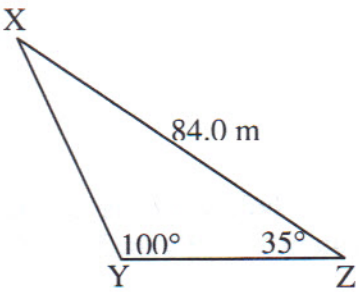
26. The half-life of sodium-24 is 15h.
a) What amount of a 4g sample remains after 30 h?
b) What amount remains after 5h, to the nearest tenth of a gram?
c) How long will it take to decay to 0.5 g?

27. A tennis ball is dropped from a height of 10m. Each time the ball touches the ground, it bounces up to about 45% of the maximum height of the previous bounce. Determine its height after 5 bounces.

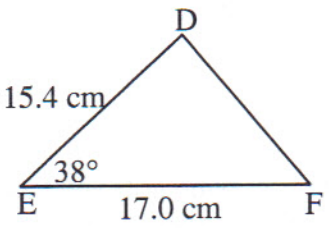
Unit 5 – Trigonometry

28. Solve each of the following triangles.

a)



b)



29. What are the solutions to the following equations, for the domain $0° \le \theta \le 360°$? Give your answers to the nearest tenth of a degree.

a) $\cos\theta = 0.1254$ b) $\tan\theta = -\frac{8}{3}$

30. The point P(-3, - 1) is on the terminal arm of an angle θ in standard position
a) Sketch and label the principal angle.
b) Determine the three primary trigonometric ratios, exactly.
c) Determine the measure of the principal angle to the nearest degree.
d) Determine the measure of the related acute angle to the nearest degree.

31. In ΔABC, $\angle C = 38.7°$, $c = 5$ cm, and $b = 12.5$ cm. Draw and label a diagram then determine the number of solutions.

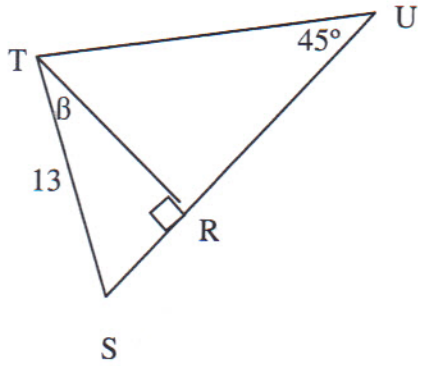
32. Solve each of the following triangles. *Draw and label a diagram first.*
a) ΔDEF: $\angle D = 31°$, $\angle E = 68°$, $d = 18$cm
b) ΔABC: $\angle A = 120°$, $b = 70$m, $c = 60$ m
c) ΔHIJ: $\angle H = 60°$, $h = 19$mm, $i = 23$mm

33. Determine the **exact value** of each of the following trigonometric ratios. *Draw and fully label a diagram* of your information, first.
a) $\sin 315°$ b) $\tan 60°$ c) $\cos 150°$

34. Draw special triangles and use them to solve each of the following for $0° \le \theta \le 360°$. Show all steps and all possible solutions.
a) $\tan\theta = \sqrt{3}$ b) $\cos\theta = -\frac{1}{2}$ c) $\sin\theta = -\frac{1}{\sqrt{2}}$

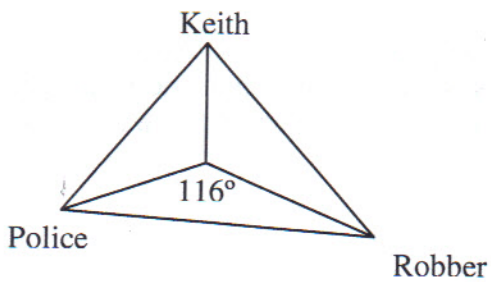
35. Determine the exact value for $1 - \frac{\sin 45°}{\cos 45°}$ using the trigonometric ratios for special angles.

36. Determine the exact area of the ΔSTU given that $\cos\beta = \frac{\sqrt{3}}{2}$:



37. Melanie is playing golf and hits the ball from the tee towards the hole, however, the ball veers to the left 21°. The tee is 295 m from the hole and she walks 156 m to where the ball lands.
a) Draw a picture to illustrate this situation.
b) How far does she need to hit the ball to get it in the hole? (Round to one decimal place).

38. Keith is looking out the window of his condo that is 166m above ground. He sees a robbery take to his left at an angle of depression of 40° and he sees a police officer to his right at an angle of depression of 34°. If the angle between the two people seen is 116°, find how far apart the police officer is from the robbery.



39. Prove the following trigonometric identities.

a) $\sin x \cos x = \dfrac{\tan x}{1+\tan^2 x}$

b) $\sin^2\theta + \cos^2\theta = (\sin\theta)(\csc\theta)$

c) $\tan^2\theta + \sin^2\theta = (\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

Unit 6 – Sinusoidal Functions

40. If $f(\theta) = 3\cos(3\theta + 180°) - 1$, name:

a) the amplitude

b) the period

c) the phase shift

d) the vertical displacement

41. If $f(\theta) = -3\sin(\frac{1}{2}\theta - 30°) + 4$, name:

a) the amplitude

b) the number of cycles in domain 0° to 360°

c) the phase shift

d) the vertical displacement

42. If $f(\theta) = -2\cos(3\theta + 90°) - 1$, where $0° \le \theta \le 360°$,

a) name the transformations necessary to sketch this function, in the correct order.

b) sketch the function by first sketching the base function in the given domain and applying each transformation.

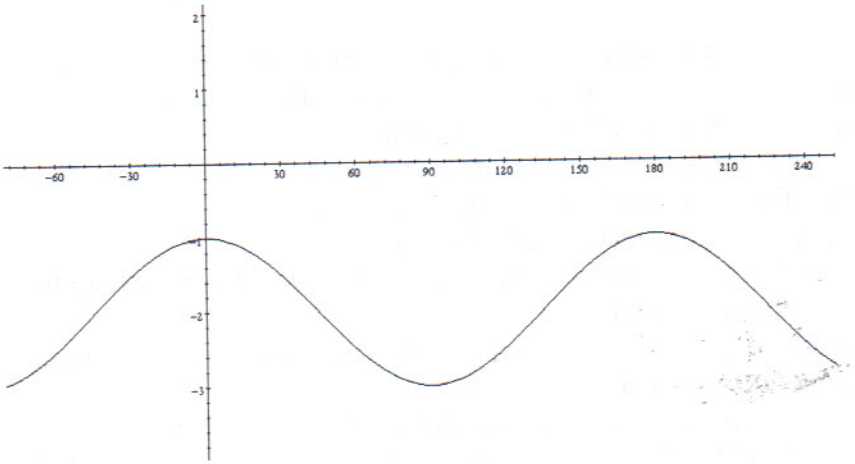
c) State the domain and range of the transformed function.

43. Sketch one cycle of the graph of $f(\theta) = 3\sin\theta$ starting at (0, 0), $\theta \ge 0$. State the domain and range.

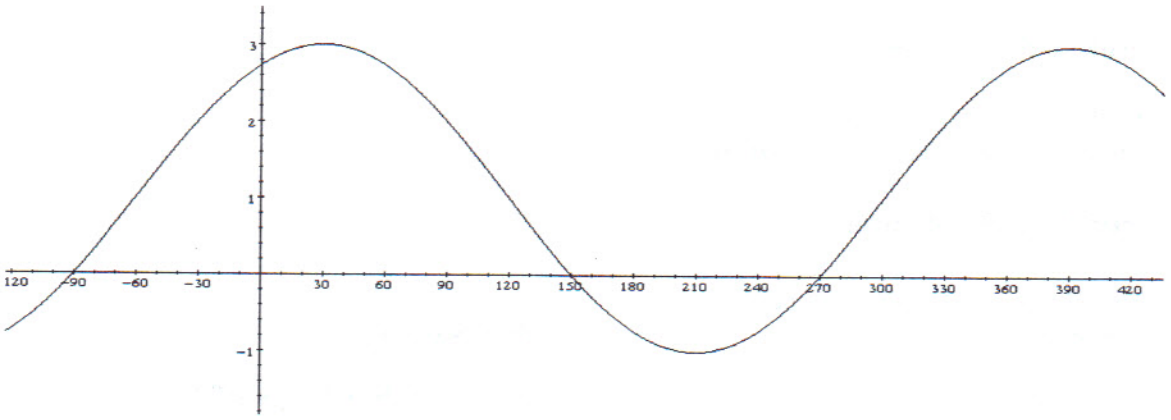
44. Sketch one cycle of the graph of $f(\theta) = \cos 2\theta$ starting at (0, 0), $\theta \ge 0$. State the domain and range.

45. Determine the equation of each of the graphs below.

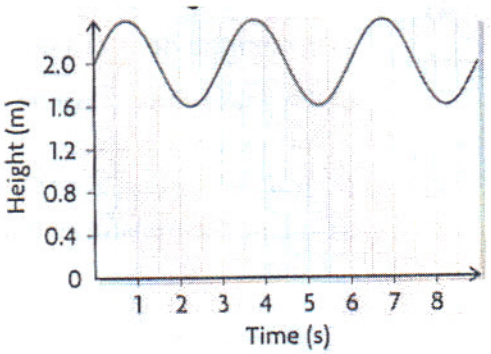
a)



b)



46. The graph on the right depicts Martha's height above the bottom of the wave pool as she bobs in the water.



a) What is the amplitude of the function? What does it represent in this situation?

b) What is the equation of the axis? What does it represent in this situation?

c) What is the period of this function? What does it represent in this situation?

d) Determine a reasonable domain and range of this function if the pool is open from 2-4 o'clock?

47. The relationship between the height of a swing in metres and time in seconds is modelled by the equation $h(t) = 1.2\sin(120t)° + 1.5$.

a) Determine the minimum and maximum heights of the swing.
b) Determine the period of the swing's motion.
c) When will a rider be at a height of 2m for the first time?
d) During the first 10 seconds, when will the rider be at a height of 1.5m.

48. Mrs. Carney is sitting in a rocking chair. The distance, $d(t)$ between the wall and the rear of the chair varies sinusoidally with time, t. At $t = 1$ second, the chair is closest to the wall at 15cm. At $t = 3$ seconds, the chair is farthest from the wall at 28cm.

a) What is the period of the function and what does it represent in this situation?
b) How far is the chair from the wall when no one is rocking in it?
c) Does Mrs. Carney start rocking forward or backward?
d) If Mrs. Carney rocks back and forth 40 times only, what is the domain of the function.
e) What is the range of the function in d)?
f) What is the amplitude of the function and what does it represent in this situation?
g) What is the equation of the sinusoidal function?
h) What is the distance between the wall and the chair at $t = 8$ seconds?
i) How long does it take for the chair to be 20cm from the wall on Mrs. Carney's 12^{th} rock back and forth?

49. Demand for potted tulips is known to follow a sinusoidal pattern. The sales data, in thousands of dollars is recorded in the chart below.

Month	J	F	M	A	M	J	J	A	S	O	N	D	J
Sales ($000)	100	125	144	150	143	126	101	75	57	50	57	75	100

a) Graph the data and draw the curve of best fit. Consider January = 0.
b) Determine a trigonometric model that best fits this data.
c) Use your equation to determine the name of the month when the sales first reach $95,000 in the year.
d) Use your equation to determine the sales in month 38.

UNIT 7 – Discrete Functions

50. Identify each of the following sequences as either arithmetic, geometric or neither, and give the values for d or r.

a) -36, 12, -4, $\frac{4}{3}$, …

b) $\frac{1}{8}, \frac{1}{2}, \frac{7}{8}, 1\frac{1}{4}, 1\frac{5}{8}, \ldots$

c) 4.0, 6.5, 9.0, 11.5, 14.0

d) 36, 18, 9, 4.5, 2.25

51. Determine the general term, in simplest form, for each of the following.

a) 5, 15, 45, 135, …

b) 5, $\frac{45}{8}, \frac{25}{4}, \frac{55}{8}, \ldots$

52. For a geometric sequence, $t_4 = 12$ and $t_7 = 96$. Determine the formula for the general term.

53. Olivia borrows $5,000 from her parents to buy a used car. They charge her 4%/a simple interest. She pays them back $5,700. How many years did it take her to repay the loan.

54. Fill in the chart below.

	A	P	i	n	r	t	Compounded
a)		$3000			6%	3.5	Quarterly
b)	$9 000				4%	6	Monthly
c)		$2500	0.015	20			Semi-annually

55. Justin invests $500 in a Canada Savings Bond which pays 3.65%/a compounded daily. How much interest does he earn over 5 years?

56. Blane wants to have $3,000 to backpack through Europe in 3 years. Blane offers to loan his father some money if he agrees to pay him interest of 3.2% compounded quarterly. How much would Blane have to loan his dad now to ensure he has enough money to go to Europe in 3 years?