Extra Practice: Using Logarithmic Laws and Properties

1. Using each expression by first using the laws of logarithms.

a) $log_2 320 - log_2 20$ b) $log_2 144 - log_2 9$
c) $log_6 4 + log_6 9$ d) $log 4 + log 25$
e) $log_8 16 + log_8 32$ f) $log_3 27 + log_3 9$

2. Use the laws of logarithms to expand each expression.

a) $log_2(14 \times 9)$ b) $log_5\left(\frac{735}{40}\right)$
c) $log_5\sqrt{25}$ d) $log_6(9 \times 8 \times 7)$
e) $log_3(15)^4$ f) $log_4\left(\frac{81}{30}\right)$

3. Evaluate each expression without using a calculator.

a) $log 25 + log 4$ b) $log_3 18 - log_3 6$
c) $log_8(8)^3$ d) $log_3\sqrt{9}$
e) $log_6 3 + log_6 12$ f) $2log_5 15 - log_5 9$
g) $log_4 32 - log_4 2$ h) $log_2(32)^4$

4. Evaluate using the law of logarithm of powers.

a) $log_3\sqrt[3]{9}$ b) $log_3\sqrt[4]{27}$
c) $log_6\sqrt[3]{36}$ d) $log_5\sqrt{125}$
e) $log_8\sqrt[3]{64}$ f) $log_4\left(\frac{1}{16}\right)$

5. Evaluate.

a) $log_8 6 - log_8 3 + log_8 2$ b) $log_2\sqrt[3]{32}$
c) $log_3 54 + log_3\left(\frac{3}{2}\right)$ d) $log_5\sqrt[5]{125}$
e) $log_8 2 + 3log_8 2 + \left(\frac{1}{2}\right)log_8 16$ f) $log_2\left(\frac{1}{8}\right)$

6. Express as a single logarithm.

a) $3log_5 2 + log_5 7$
b) $2log_3 8 - 5log_3 2$
c) $2log_2 3 + log_2 5$
d) $log_3 12 + log_3 2 - log_3 6$
e) $log_4 3 - log_4 2 + \left(\frac{1}{2}\right)log_4 8$
f) $2log 8 + log 9 - log 36$

7. Evaluate $log_2(8 \times 32) + log_7\left(49(\sqrt[4]{7})\right)$.

8. Given $x = log_2 5$ and $y = log_2 3$, evaluate each expression in terms of x and y.

a) $log_2 15$ b. $log_2 0.6$ c. $log_2 125$

9. Solve for x.

a) $log_2 x = log_2 5 + log_2 10$ b) $log_3 x = log_3 18 - log_3 3$
c) $log x = log 84 + log 5 - log 7$ d) $log x = 2log 4 + 3log 3$
e) $log_5 x - log_5 8 = log_5 6 + 3log_5 2$

10. Express as a single logarithm. Assume all variables are positive.

a) $log_2 x + log_2 y + log_2 z$ b) $log_5 u - log_5 v + log_5 w$
c) $log_6 a - (log_6 b + log_6 c)$ d) $log_2 x^2 - log_2 xy + log_2 y^2$
e) $1 + log_3 x^2$ f) $3log_4 x + 2log_4 x - log_4 y$

11. If $log_3 x = 0.2$, find the value of $log_3 x\sqrt{x}$.

12. If $log_a w = \frac{1}{2}log_a x + log_a y$, exprss w in terms of x and y.

13. COMMUNCIATION: Explain the similarities between the laws of exponents and the laws of logarithms.

14. Use a calculator to evaluate each expression to two decimal places.

a) $log 4^8$ b) $log\sqrt{40}$ c) $log 9^4$
d) $log 200 \div log 50$ e) $(log 20)^2$ f) $5log 5$

15. APPLICATION: The loudness, L, of a sound is related to the sound's intensity, I, by $L = 10log\frac{I}{I_0}$ where L is measured in decibels, I is measured in watts per square metre, and I_0 is the intensity of a barely audible sound. By how many decibels does the loudness increase if the intensity if the intensity of the sound from a tuning fork is tripled?

16. A barely audible sound has an intensity of $I_0 = 10^{-12}\ W/m^2$. Use the formula in question 15 to calculate the loudness of each sound :

a) A falling pin: $I = 10^{-11}\ W/m^2$ b) Quiet conversation: $I = 10^{-6}\ W/m^2$
c) Subway: $I = 10^{-3}\ W/m^2$ d) Jet at take-off: $I = 1\ W/m^2$

Answers

1. (a) 4 (b) 4 (c) 2
   (d) 2 (e) 3 (f) 1
2. (a) $\log_2 14 + \log_2 9$ (b) $\log_5 735 - \log_5 40$
   (c) $\frac{1}{2}\log_7 25$ (d) $\log_6 9 + \log_6 8 + \log_6 7$
   (e) $4 \log_3 15$ (f) $\log_4 81 - \log_4 30$
3. (a) 2 (b) 1 (c) 9 (d) 1
   (e) 2 (f) 2 (g) 2 (h) 20
4. (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$
   (d) $\frac{3}{2}$ (e) $\frac{2}{3}$ (f) -2
5. (a) $\frac{2}{3}$ (b) $\frac{5}{3}$ (c) 4
   (d) $\frac{2}{5}$ (e) 2 (f) -3
6. (a) $\log_5 56$ (b) $\log_3 2$ (c) $\log_2 45$
   (d) $\log_3 4$ (e) $\log_4 3\sqrt{2}$ (f) $\log 16$
7. $\frac{41}{4}$
8. (a) $x + y$ (b) $y - x$ (c) $3x$
9. (a) $x = 50$ (b) $x = 6$ (c) $x = 60$ (d) $x = 432$ (e) $x = 384$
10. (a) $\log_2 xyz$ (b) $\log_5 \frac{tnw}{v}$ (c) $\log_6 \frac{a}{bc}$
    (d) $\log_2 xy$ (e) $\log_3 3x^2$ (f) $\log_4 \frac{x^5}{y}$
11. 0.3
12. $w = y\sqrt{x}$
13. $\log MN = \log M + \log N$ $a^M a^N = a^{M+N}$
    $\log \frac{M}{N} = \log M - \log N$ $\frac{a^M}{a^N} = a^{M-N}$
14. (a) 4.82 (b) 0.80 (c) 3.82
    (d) 1.35 (e) 1.69 (f) 3.49
15. $10 \log 3$
16. (a) 10 dB (b) 60 dB (c) 90 dB (d) 120 dB