MHF 4U Date Key

Extra Practice: Using Laws of Logarithms

$y = \log_a x \Leftrightarrow a^y = x$

$\log_a mn = \log_a m + \log_a n$

$\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_a (m^n) = n \log_a m$

$\log_a \left(m^{\frac{1}{n}} \right) = \frac{1}{n} \log_a m$

$\log_a (a^m) = m$

$a^{\log_a m} = m$

1. Express as a single logarithm. Then, evaluate.

a) $\log_{10} 8 + \log_{10} 1.25$

$= \log_{10}(8 \times 1.25)$

$= \log_{10} 10$

$= 1$

b) $\log_2 80 - \log_2 5$

$= \log_2\left(\frac{80}{5}\right)$

$= \log_2 16$

$= 4$

c) $\log 40 + \log 5 - \log 2$

$= \log\left(\frac{40 \times 5}{2}\right)$

$= \log 100$

$= 2$

d) $\log_7 245 + \log_7 \frac{1}{5}$

$= \log_7 \left(245 \times \frac{1}{5}\right)$

$= \log_7 49$

$= 2$

e) $\log_3 \sqrt[3]{9}$

$= \log_3 \sqrt{3^2}$

$= \log_3 (3^2)^{1/2}$

$= \log_3 3$

$= 1$

f) $\log_3 \sqrt[5]{27} - \log_3 \sqrt[5]{2187}$

$= \log_3 27^{1/5} - \log_3 (2187)^{1/5}$

$= \frac{1}{5} \log_3 \left(\frac{27}{2187}\right)$

$= \frac{1}{5} \log_3 \frac{1}{81}$

$= \frac{1}{5}(-4) = -4/5$

2. Evaluate.

a) $2 \log_3 12 - 2 \log_3 4$

$= \log_3 12^2 - \log_3 4^2$

$= \log_3 144 - \log_3 16$

$= \log_3 \left(\frac{144}{16}\right)$

$= \log_3 9$

$= 2$

b) $\log_4 24 + \log_4 \frac{64}{3} - \log_4 32$

$= \log_4 \left(24 \times \frac{64}{3}\right) - \log_4 32$

$= \log_4 \left(\frac{512}{32}\right)$

$= \log_4 16$

$= 2$

c) $\log_5 \sqrt{225} - \log_5 \sqrt{9}$

$= \log_5 \sqrt{\frac{225}{9}}$

$= \log_5 \sqrt{25}$

$= \frac{1}{2} \log_5 25$

$= \frac{1}{2}(2) = 1$

d) $\frac{1}{2} \log_3 225 - \log_3 5 + 3 \log_3 3$

$= \log_3 \frac{\sqrt{225} \times 3^3}{5}$

$= \log_3 \left(\frac{15 \times 27}{5}\right)$

$= \log_3 81$

$= 4$

3. Expand using laws.

a. $\log_6 x^2 y^3$

$= \log_6 x^2 + \log_6 y^3$

$= 2 \log_6 x + 3 \log_6 y$

b. $\log_2 \frac{\sqrt[4]{a}}{bc}$

$= \frac{1}{4} \log_2 a - \log_2 b - \log_2 c$

c. $\log_b \frac{a^2 c}{\sqrt{d}}$

c) $2 \log_b a + \log_b c - \frac{1}{2} \log_b d$

MHF 4U Date________________________________

Recall the logarithmic function, $f(x) = \log_a x$ has a vertical asymptote at $x = 0$ with restriction $x > 0$. This implies all logarithmic expressions and equations have restrictions on their domain and must be stated.

4. Write as a single logarithm and state restrictions.

a. $3log_4x - \frac{1}{2}log_4x + 2log_4y$

$= \log_4 x^3 - \log_4 \sqrt{x} + \log_4 y^2$

$= \log_4 \left(\frac{x^3}{\sqrt{x}} \cdot y^2\right)$

$= \log_4 (x^{5/2} y^2)$

b. $\log(x + 1) + 2\log(2x + 1) - \log(x - 3)$

$= \log\left[\frac{(x+1)(2x+1)^2}{(x-3)}\right]$

5. Simplify and state restrictions on :

a. $\log(x^3 - 27) - \log(x - 3)$

$= \log \frac{(x^3-27)}{(x-3)}, \quad x > 3$

$= \log \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}}$

$= \log(x^2+3x+9)$

b) $\log(x + 2) + log\,(x - 2)$

$= \log(x+2)(x-2), \quad x > 2$

This new formula will be proven. For now, we will accept it.

Change in Base Rule

$$log_ab = \frac{logb}{loga}$$

6. Use a calculator to evaluate the following correct to four decimal places.

a) $\log 2372.53$

$= \frac{\log 2372.53}{\log 10}$

b) $\log_{12} 3$

$= \frac{\log 3}{\log 12}$

c) $\log_5 82 - \log_5 12.5 + \log_5 8$

$= \log_5\left(\frac{82 \times 8}{12.5}\right)$

$= \log_5 52.48$