The Limit of a Function
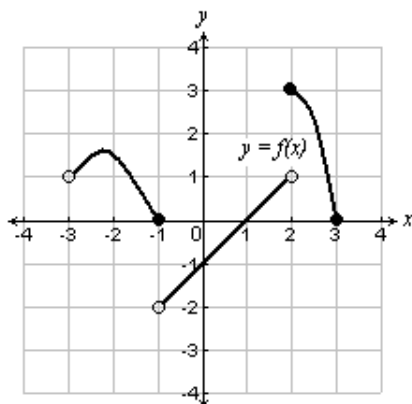
1. Use the graph of $f(x)$ to state the value of each of the following, if it exists. If it does not exist, explain why.

a) $\lim\limits_{x \to -3^+} f(x)$ b) $\lim\limits_{x \to -1^-} f(x)$ c) $\lim\limits_{x \to -1^+} f(x)$

d) $\lim\limits_{x \to -1} f(x)$ e) $\lim\limits_{x \to 1^-} f(x)$ f) $\lim\limits_{x \to 1^+} f(x)$

g) $\lim\limits_{x \to 1} f(x)$ h) $f(1)$ i) $\lim\limits_{x \to 2^-} f(x)$

j) $\lim\limits_{x \to 2^+} f(x)$ k) $\lim\limits_{x \to 2} f(x)$ l) $f(2)$

m) $\lim\limits_{x \to 3^-} f(x)$ n) $f(3)$

2. Let
$$f(x) = \begin{cases} 3 - x^2 & \text{if } x \in (-\infty, 1) \\ x + 1 & \text{if } x \in [1, \infty) \end{cases}$$
a) Graph the function $f(x)$.

b) Find each limit, if it exists. If the limit does not exist, explain why.
   i) $\lim\limits_{x \to 1^-} f(x)$ ii) $\lim\limits_{x \to 1^+} f(x)$ iii) $\lim\limits_{x \to 1} f(x)$

c) Is the function continuous for all values of x? Explain.

3. Let
$$f(x) = \begin{cases} x^2 & \text{if } x \in (-\infty, -2) \\ -2x & \text{if } x \in [-2, 2] \\ 2x - 4 & \text{if } x \in (2, \infty) \end{cases}$$
a) Graph the function $f(x)$.

b) Find each limit, if it exists. If the limit does not exist, explain why.
   i) $\lim\limits_{x \to -2^-} f(x)$ ii) $\lim\limits_{x \to -2^+} f(x)$ iii) $\lim\limits_{x \to -2} f(x)$
   iv) $\lim\limits_{x \to 2^-} f(x)$ v) $\lim\limits_{x \to 2^+} f(x)$ vi) $\lim\limits_{x \to 2} f(x)$

c) Is the function continuous for all values of x? Explain.