

First Differences

Consider the following relations.

- (i) Find the first differences to determine a pattern in the given table of values.
- (ii) Graph the relation. What do you notice about the relation? Is there a connection between the value of the first differences and the graph drawn? If so, justify your reasoning.
- (iii) Using the above information, determine an equation that describes the relation.
- (iv) Is there a connection between the 1st differences, the graph and the equation? Explain.

(A) Relation 1

First Differences
 Differences between consecutive y-values with evenly spaced x-values.
 In this example, the x-values increase by a value of 1.

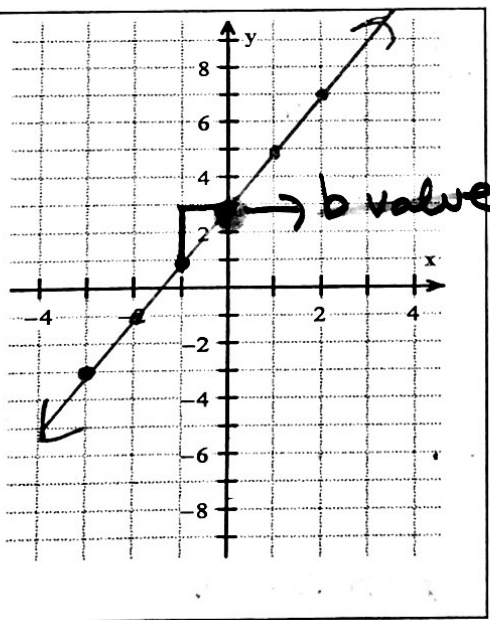
x	y	First Differences
-3	-3	
-2	-1	$-1 - (-3) = -1 + 3 = 2$
-1	1	$1 - (-1) = 1 + 1 = 2$
0	3	$3 - 1 = 2$
1	5	$5 - 3 = 2$
2	7	$7 - 5 = 2$

i) For every value of x that it increases by 1 the 1st difference equals 2.

ii) • Relation is a linear
 • Positive correlation
 $m = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

Subtract y-values

b-value



Notice
 (ii) The 1st difference is equivalent to the slope. ($\Delta x = 1$)

∴ $1st\ diff. = m$

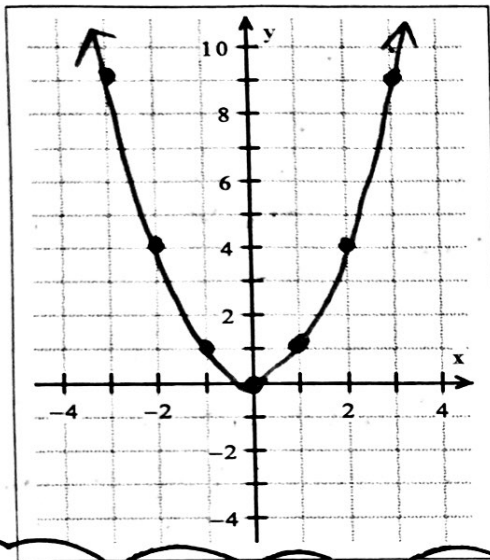
(iii) $y = mx + b$
 $y = 2x + 3$

(B) Relation 2

x	y	First Differences
-3	9	
-2	4	$4-9 = -5$
-1	1	$1-4 = -3$
0	0	$0-1 = -1$
1	1	$1-0 = 1$
2	4	$4-1 = 3$
3	9	$9-4 = 5$

i) 1st differences are not the same

ii) Relation is a curve. Graph is non-linear.



iii) $y = x \times x$
 $y = x^2$

iv) A non-linear relation does not have an equation in the form $y = mx + b$

Summary

(1) If the 1st differences are constant (the same) then the relation is linear and its equation is in the form of $y = mx + b$.

(2) If the 1st differences are non-constant, then the relation produces a curve which is non-linear.

Example: Refer to each equation. Is the relation linear or non-linear? Justify.

(i) $y = -4x + 5$ linear $m = -4$, $b = 5$, $y = mx + b$

(ii) $y = 6^x$ non-linear x is an exponent

(iii) $y = \frac{2}{3}x - \frac{6}{7}$ linear $-x$ is in the denominator

(iv) $y = \frac{9}{x}$ non-linear $-x$ is in the denominator

(v) $y = -2x^2 + 6$ non-linear exponent 2

Learning Goal:

- I can find the 1st differences in a table of values and determine if the relation is linear.
- I can determine a relation is linear using its equation.