**Double Angle Formulas**

Written below are the three double angle identities for sine, cosine and tangent. Note there are 3 identities for $cos2x$. There is no need to memorize all three of them, as the last two can always be determined by using the Pythagorean identity $sin^{2}x+cos^{2}x=1$ and isolating for either $sin^{2}x$ or $cos^{2}x$.

|  |  |
| --- | --- |
| $$sin2x=2sinxcosx$$ | $$cos2x=\left\{\begin{array}{c}cos^{2}x-sin^{2}x\\ \\1-2sin^{2}x\\\\2cos^{2}x-1\end{array}\right.^{}$$ |
| $$tan2x=\tan(\left(x+x\right))=\frac{tanx+tanx}{1-tanx(tanx)}$$ |

Before we start working with these identities, let’s demonstrate why these work.

**Ex 1**: Let $x=60°$. Show that the above identities are true.

*Note:* $sin2x\ne 2sinx$ *; similarly* $cos2x\ne \frac{1}{2}cosx$

**Ex 2:** Write an equivalent expression for the following using a double identity.

a) $cos4x$ b) $sin6x$ c) $cos\left(\frac{2}{3}x\right)$

**Ex 3:** Express as a single cosine or sine function.

a) $8 sin2x cos2x$ b) $1-2sin^{2}\left(\frac{3}{2}x\right)$ c) $10cos^{2}\left(\frac{1}{3}x\right)-5$

**Ex 4:** If $sinx=\frac{4}{5}$ , $\frac{π}{2}<x<π$ , find the value of $sin2x.$

**Ex 5**: If $cosx=\frac{2}{3}$ , find the value of $cos4x$. What quadrant does 4x lie in?

**Ex 6:** Evaluate using exact values.

 a) $sin\left(\frac{9π}{8}\right)$ b) $cos\left(-\frac{5π}{8}\right)$

Half Angle Formulas

We know $cos2x=2cos^{2}x-1$ so that $cosx=2cos^{2}\left(\frac{1}{2}x\right)-1$.

Isolating for $cos\left(\frac{1}{2}x\right)$ we get:

$$cosx+1=2cos^{2}\left(\frac{1}{2}x\right)$$

$$\frac{cosx+1}{2}=cos^{2}\left(\frac{1}{2}x\right)$$

$$\pm \sqrt{\frac{cosx+1}{2}}=cos\left(\frac{1}{2}x\right)$$

**Ex 7:** Evaluate $cos\left(\frac{7}{12}π\right)$ using exact values.

**Ex 8:** Find the exact values of $cos\left(\frac{x}{2}\right)$ if $sinx=-\frac{3}{5}$ where $π<x<\frac{3}{2}π$. What quadrant does the angle lie?

**Ex 8:** Prove $sin\left(\frac{1}{2}x\right)=\pm \sqrt{\frac{1-cosx}{2}}$

**Ex 9:** Prove $tan\left(\frac{1}{2}x\right)=\frac{sinx}{1+cosx}$



**Ex 10:**