## Transformations of functions

**Part 1:** Vertical translation.

1. Consider . Draw the graphs of  and  *.* Determine Domain and Range for each function.

*x*

*y*



|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| *f*(*x*)  *y=* |  |  |
| *f*(*x*) *+ 3*  *y=* |  |  |
| *f*(*x*) *– 2*  *y=* |  |  |

**a.** How could the graph of  be obtained from the graph of  ?

0

**b.** How could the graph of  be obtained from the graph of  ?

**c.** How could the graph of  be obtained from the graph of  ? (*c* is any positive real

number)

**d.** How could the graph of  be obtained from the graph of  ? (*c* is any positive real

number)

This transformation is called a **vertical translation/shift**

Draw the graphs of  and  for each *f*(*x*) *.* Determine Domain and Range for each function.

**a)**

|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| *f*(*x*)  *y=* |  |  |
| *f*(*x*) *+ 3*  *y=* |  |  |
| *f*(*x*) *– 2*  *y=* |  |  |



**b)**

|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| *f*(*x*)  *y=* |  |  |
| *f*(*x*) *+ 3*  *y=* |  |  |
| *f*(*x*) *– 2*  *y=* |  |  |

**c)**



|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| *f*(*x*) |  |  |
| *f*(*x*) *+ 3* |  |  |
| *f*(*x*) *– 2* |  |  |

**Part 2:** Horizontal translation.

1. Consider . Draw the graphs of  and  *.* Determine Domain and Range for each function.

*x*

*y*



|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| *f*(*x*)  *y=* |  |  |
| *y=* |  |  |
| *y=* |  |  |

**a.** How could the graph of  be obtained from the graph of  ?

**b.** How could the graph of  be obtained from the graph of  ?

**c.** How could the graph of  be obtained from the graph of  ? (*d* is any positive real

number)

**d.** How could the graph of  be obtained from the graph of  ? (*d* is any positive real

number)

This transformation is called a **horizontal translation**

Draw the graphs of  and  for each *f*(*x*) *.* Determine Domain and Range for each function.



|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| *f*(*x*)  *y=* |  |  |
| *f*(*x+2*)  *y=* |  |  |
| *f*(*x - 3*)  *y=* |  |  |



|  |  |  |
| --- | --- | --- |
| Function | Domain | Range |
| *f*(*x*)  *y=* |  |  |
| *f*(*x+2*)  *y=* |  |  |
| *f*(*x - 3*)  *y=* |  |  |

**Part 3:** Combination of Horizontal and Vertical translations.

Draw the graphs of  for each *f*(*x*) *.* Determine Domain and Range for each function.



**Part 4**: Horizontal stretch/compression.



|  |  |
| --- | --- |
|  | |
| *x* | *f*(*x*) |
| - 6 |  |
| - 3 |  |
| 0 |  |
| 3 |  |
| 6 |  |

|  |  |
| --- | --- |
|  | |
| *x* | *f*(*x*) |
| - 1 |  |
| - 1/2 |  |
| 0 |  |
| 1/2 |  |
| 1 |  |

**a.** How could the graph of  be obtained from the graph of  ?

**b.** How could the graph of  be obtained from the graph of  ?

**c.** How could the graph of  be obtained from the graph of  ? Consider cases when *k* > 1

and when 0 < *k* < 1

**Part 5:** Reflections of Functions

1. Graph  on each grid.

*x*

*y*



1. Complete the table of values for and graph it. State its domain and range.

|  |  |
| --- | --- |
| x | y |
| 0 |  |
| 1 |  |
| 4 |  |
| 9 |  |
| 16 |  |

1. Complete the table of values for and graph it. State its domain and range.

*x*

*y*



|  |  |
| --- | --- |
| x | y |
| 0 |  |
| - 1 |  |
| - 4 |  |
| - 9 |  |
| - 16 |  |

The graph of  which is the same as  is the graph of 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

The graph of which is the same as  is the graph of 

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

*Points that are unaltered by a transformation are known as* ***invariant.***

## Reflections of Functions Practice

1. Complete the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| *Given f(x), sketch…* |  |  |  |
| *Given f(x), sketch…* |  |  |  |
| *Given f(x), sketch…* |  |  |  |
| *Given f(x), sketch…* |  |  |  |

1. In each graph below,  is indicated and drawn with a thin line. Give the correct function notation for the second function – drawn with the thicker line.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| **a)** | **b)** | **c)** | **d)** |
|  |  |  |  |
| **e)** | **f)** | **g)** | **h)** |

1. Sketch each of the following functions using transformations.
2. ** b) **

*x*

*y*



*x*

*y*



**Part 5**: Vertical stretch/compression.



Sketch each of the function s below using the step-pattern method.

|  |
| --- |
|  |
| Coordinates of the vertex  Direction of opening  Step pattern |

|  |
| --- |
|  |
| Coordinates of the vertex  Direction of opening  Step pattern |

**a.** How could the graph of  be obtained from the graph of  ?

**b.** How could the graph of  be obtained from the graph of  ?

**c.** How could the graph of  be obtained from the graph of  ? Consider cases when *k* > 1

and when 0 < *k* < 1

## Vertical stretches/compression of Functions Practice

1. Sketch each of the following functions using transformations.
2. ** (b) **

*x*

*y*



*x*

*y*



1. Sketch each of the following functions using transformations.
2. ** (b) **

*x*

*y*



*x*

*y*

