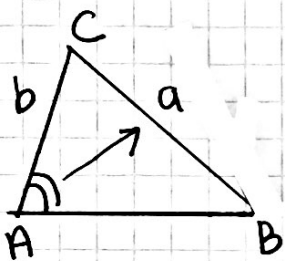


## SINE LAW: AMBIGUOUS CASE

The Ambiguous case occurs in a  $\Delta ABC$  when you attempt to use the SINE LAW given a  $\Delta$  in the form SSA.

This law states there are 4 possibilities to draw a  $\Delta$  in the form SSA.

(A) If  $\angle A$  is acute ( $< 90^\circ$ )

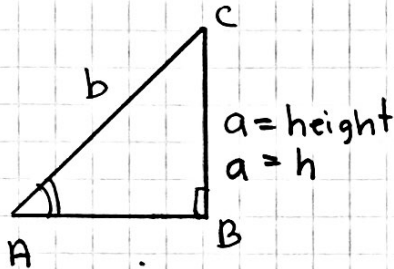


Fixed

- 1  $\Delta$  can be drawn

if  $a \geq b$

CASE (1)



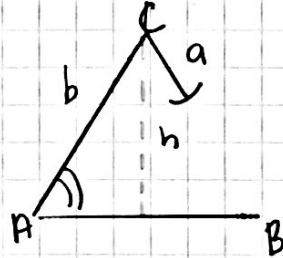
$a = \text{height}$   
 $a = h$

$$\frac{a}{b} = \sin A$$

$$h = b \sin A$$

- 1  $\Delta$  can be drawn but it is a right  $\Delta$
- if  $h = a = b \sin A$   
then  $\angle B = 90^\circ$

CASE (2)

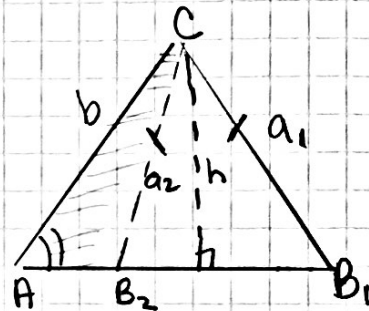


if  $a < h$   
 $a < b \sin A$

then a  $\Delta$  cannot be drawn

- No solution
- $\angle B$  does not exist

CASE (3)



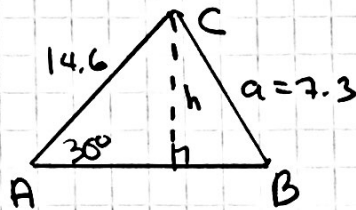
- 2 solutions
- a  $\Delta$  can be drawn 2 different ways
- a is longer than h and smaller than b

if  $h < a < b$

CASE (4)

Examples (1)

ie:  $a = 7.3$ ,  $b = 14.6$  &  $A = 30^\circ$   
Find missing side



SSA - test ambiguity

$$\frac{h}{14.6} = \sin 30^\circ$$

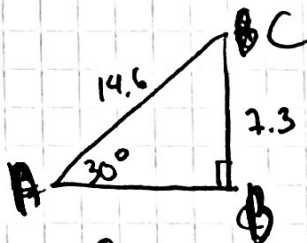
$$h = 14.6 \sin 30^\circ$$

$$h = 14.6 \left(\frac{1}{2}\right)$$

$$= 7.3$$

$$= a$$

$$\therefore h = a$$

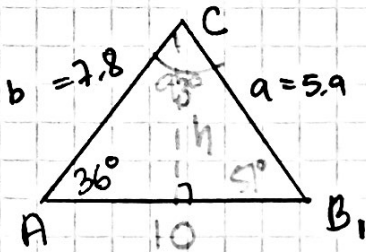


$\therefore$  only 1 Right  $\Delta$

can be drawn

Example (2)

$$a = 5.9, b = 7.8, \angle A = 36^\circ$$



$\Rightarrow$  SSA test ambiguity

$$\frac{b}{\sin 36^\circ} = \frac{a}{\sin B}$$

$$h = 7.8 \sin 36^\circ$$

$$h = 4.6$$

$$h < a < b$$

$$4.6 < 5.9 < 7.8$$

2 solns  
2  $\Delta$ 's

Solution 1

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{7.8} = \frac{\sin 36^\circ}{5.9}$$

$$\sin B = \frac{7.8 \sin 36^\circ}{5.9}$$

$$\angle B_1 = 51^\circ$$

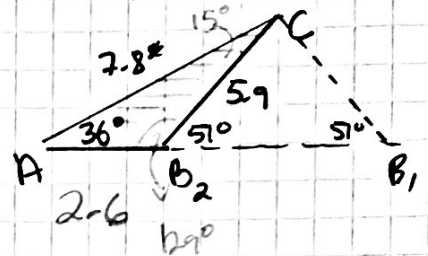
$$\angle C_1 = 180^\circ - 51^\circ - 36^\circ = 93^\circ$$

$$\frac{c}{\sin 93^\circ} = \frac{5.9}{\sin 36^\circ}$$

$$c_1 = \frac{5.9 \sin 93^\circ}{\sin 36^\circ}$$

$$c_1 = 10$$

Solution 2



$$\angle B_2 = 180^\circ - 51^\circ = 129^\circ$$

$$\angle C_2 = 180^\circ - 129^\circ - 36^\circ = 15^\circ$$

$$\frac{c_2}{\sin 15^\circ} = \frac{5.9}{\sin 36^\circ}$$

$$c_2 = \frac{5.9 \sin 15^\circ}{\sin 36^\circ}$$

$$c_2 = 2.6$$