1.5 PROPERTIES OF LIMITS Notes

Direct Substitution

$\lim_{x \to a} f(x) = f(a)$ The <u>limit of a polynomial</u> is the polynomial evaluated at $x = a$.	e.g. $\lim_{x \to 2}(x^2 - 3x) = 2^2 - 3(2) = -2$

Limit of a Constant, $c \in \Re$

$\lim_{x \to a} c = c$ The limit of a constant is the constant.	e.g. $\lim_{x \to 5} 13 = 13$

More Properties of Limits

Suppose that the $\lim_{x \to a} f(x)$ and the $\lim_{x \to a} g(x)$ exist and that c is a constant.

1. $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$ The limit of a sum is the sum of the limits.	$\lim_{x \to 2}\left[4x^2 + 3x\right] = \lim_{x \to 2} 4x^2 + \lim_{x \to 2} 3x$
2. $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$ The limit of a difference is the difference of the limits.	$\lim_{x \to 2}\left[4x^2 - 3x\right] = \lim_{x \to 2} 4x^2 - \lim_{x \to 2} 3x$
3. $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$ The limit of a constant times a function is a constant times the limit.	$\lim_{x \to 2}\left[4x^2 + 3x\right] = 4\lim_{x \to 2} x^2 + 3\lim_{x \to 2} x$
4. $\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x)$ The limit of a product is the product of the limits.	$\lim_{x \to 2}\left[(4x-1)(2x+3)\right] = \lim_{x \to 2}(4x-1) \times \lim_{x \to 2}(2x+3)$
5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \neq 0$ The limit of a quotient is the quotient of the limits, if the denominator is not zero.	$\lim_{x \to 2}\left[\frac{4x-1}{2x+3}\right] = \frac{\lim_{x \to 2}(4x-1)}{\lim_{x \to 2}(2x+3)}$

EX: Given $\lim_{x \to 3} f(x) = -2$ and $\lim_{x \to 3} g(x) = 1$, use the limit properties to find $\lim_{x \to 3} \frac{2f(x) + g(x)}{-4\sqrt{g(x)}}$

SUMMARY OF LIMITS - Including Indeterminate Form $\frac{0}{0}$

Method/Technique Used	Example
I. Direct Substitution ✧ Substitute into the function the value of x	a) $\lim_{x\to 2}(x^2+1)$
If direct substitution produces $\frac{0}{0}$ or any form of **Indeterminate,** then the following methods are used.	
II. Factoring ✧ Factor function, simplify then use direct substitution. ✧ $\lim_{x\to a}\frac{F(x)}{G(x)} = \lim_{x\to a}\frac{(x-a)f(x)}{(x-a)g(x)} = \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$	b) $\lim_{x\to 1}\frac{x^2-1}{x-1}$
III. Conjugate Radicals ✧ Eliminate radical by multiplying both numerator & denominator by conjugate ✧ Simplify by canceling out common factors	c) $\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$
IV. Simplification ✧ Find common denominator and simplify by canceling out common factors	d) $\lim_{t\to 0}\frac{\frac{4}{2+t}-2}{t}$
V. Graphing: One & Two Sided Limits ✧ Find the left and right limits ✧ If $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x)$ then $\lim_{x\to 1} g(x)$ exists.	e) If $g(x)=\begin{cases} x^2, if\ x\le 1 \\ 2-x, if\ x>1\end{cases}$, find $\lim_{x\to 1} g(x)$.
VI. Using Change of Variable ✧ Works well if term has a rational exponent ✧ If the change in variable is $u = g(x)$ then: $x \to a$, $u \to g(a)$	f) $\lim_{x\to 0}\frac{(1+2x)^{\frac{1}{3}}-1}{x}$ g) $\lim_{x\to a}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$
VII. Absolute Function ✧ Rewrite function as a piece-wise function. ✧ Find left and right limits ✧ If $\lim_{x\to a^-} g(x) = \lim_{x\to a^+} g(x)$ then $\lim_{x\to a} g(x)$ exists.	h) $\lim_{x\to 0}\lvert x\rvert$ i) $\lim_{x\to 4}\frac{\lvert x-4\rvert}{x-4}$ k) $\lim_{x\to a} x\lvert x\rvert$
VIII. Infinite, $x \to \pm\infty$ ✧ Compare the degree in numerator and denominator (Advanced Functions) or ✧ Divide each term by the largest power & simplify	l) $\lim_{x\to\infty}\frac{1-x^3}{1+2x^3}$
IX. Root Function ✧ Find the restriction on the domain ✧ Determine the left and right limits	m) $\lim_{x\to 4}\sqrt{4-x}$
X. Thinking	n) $\lim_{x\to\infty} 5^{-x}$
XI. Graphing a Sequence ✧ Create a visual to help determine if the sequence oscillates between values, or it approaches a	o) Find the limit $1, 0, \frac{1}{2}, 0, \frac{1}{3}, 0, \frac{1}{4}, 0, \ldots$