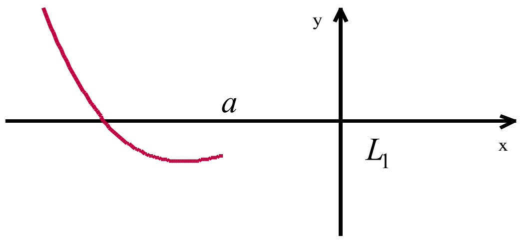
1.4 Limit of A Function - Notes

(A) Left-Hand Limit

If the values of $y = f(x)$ can be made arbitrarily close to L by taking x sufficiently close to a with $x < a$, then:

$$\lim_{x \to a^-} f(x) = L_1$$

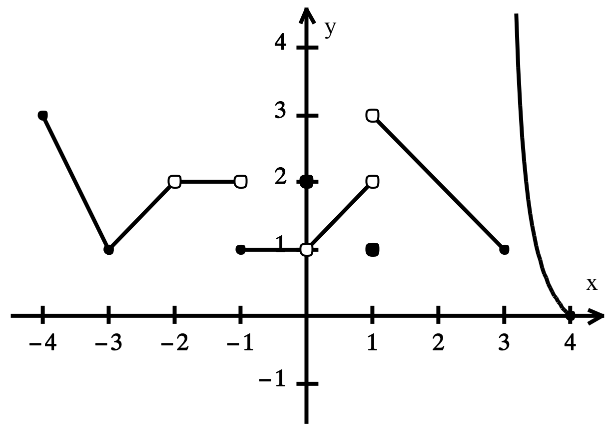
Read as: The limit of the function $f(x)$ as x approaches a from the left is L_1.



Notes:

1. The function may or may not be defined at a.
2. DNE stands for Does Not Exist
3. L_1 is a number, $L_1 \in \Re$
4. ∞ is not a number, it is an arbitrary large #

Ex.1 Use the function $y = f(x)$ defined by the following graph to find each limit.



a) $\lim_{x \to -4^-} f(x)$

b) $\lim_{x \to -2^-} f(x)$

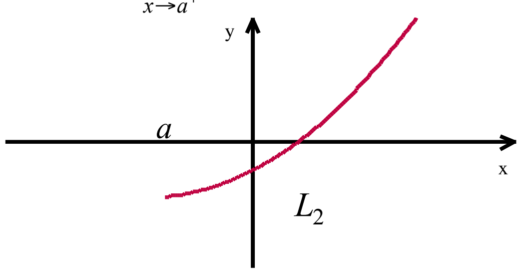
c) $\lim_{x \to -1^-} f(x)$

d) $\lim_{x \to 3^-} f(x)$

(B) Right- Hand Limit

If the values of $y = f(x)$ can be made arbitrarily close to L_2 by taking x sufficiently close to a with $x > a$, then:

$$\lim_{x \to a^+} f(x) = L_2$$



Read as: The limit of the function $y = f(x)$ as x approaches a from the right is L_2.

Notes:

1. L_2 is a number, $L_2 \in \Re$
2. The function may or may not be defined at a

Ex. 2 Use the function $y = f(x)$ defined in Ex. 1 to find each limit.

a) $\lim_{x \to -4^+} f(x)$

b) $\lim_{x \to -2^+} f(x)$

c) $\lim_{x \to 3^+} f(x)$

d) $\lim_{x \to 1^+} f(x)$

e) $\lim_{x \to 0^+} f(x)$

(C) Limits and Their Existence

If the number L is the limit of a function $y = f(x)$ as x approaches a from both the left and right side then:

$$\lim_{x \to a} f(x) = L$$

Read as: The limit of the function $f(x)$ as x approaches a is L.

Notes:

1. $\lim_{x \to a} f(x)$ may exist even if $f(x)$ is not defined.

Ex. 3 Use the function $y = f(x)$ defined in Ex. 1 to find each limit.

a) $\lim_{x \to 0} f(x)$

b) $\lim_{x \to -4} f(x)$

c) $\lim_{x \to -2} f(x)$

d) $\lim_{x \to 3} f(x)$

e) $\lim_{x \to 1} f(x)$

2. if $\lim_{x\to a} f(x) = L$ then

$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$

3. if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ then $\lim_{x\to a} f(x)\ DNE$

4. $\lim_{x\to a} f(x) = f(a)$. In this case, the graph of $f(x)$ passes through the point $(a, f(a))$, the limit of $f(x)$ exists and $f(a)$ is defined.

f) $\lim_{x\to -1} f(x)$

g) $\lim_{x\to -3} f(x)$

(D) Substitution

If the function is defined by a formula (algebraic expression) then the limit of the function at a point a may be determined by substitution.

$$\lim_{x\to a} f(x) = f(a)$$

…………….(#4 in notes must be true)……………….

Notes:

1. In order to use substitution, the function must be defined on both sides of the number a.
2. Substitution does not work if you get one of the following indeterminate cases.

$\frac{0}{0}$ $\quad \frac{\infty}{\infty}$ $\quad 0^0$ $\quad \infty^0$ $\quad 0\times\infty$ $\quad \infty-\infty$ $\quad 1^\infty$

Ex. 4 Find each limit.

a) $\lim_{x\to -1^+} \frac{x^2}{x+2}$

b) $\lim_{x\to -1^-} \frac{x^2}{x+2}$

c) $\lim_{x\to -1} \frac{x^2}{x+2}$

d) $\lim_{x\to 1^-} \sqrt{1-x}$

e) $\lim_{x\to 1^+} \sqrt{1-x}$

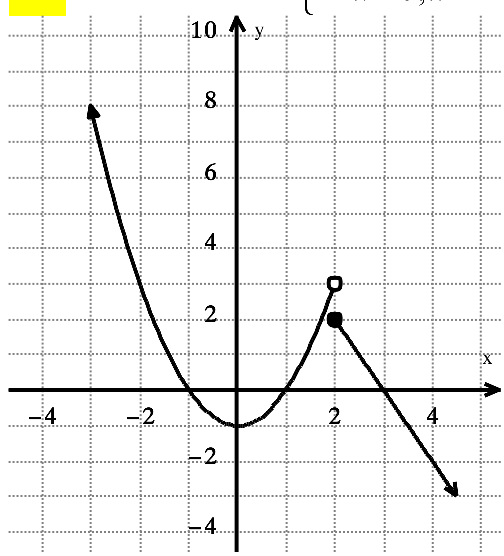
f) $\lim_{x\to 1} \sqrt{1-x}$

(E) Piece – Wise Functions

If the function changes formula at a then:

1. Sketch the function if necessary.
2. Use the appropriate formula to find first the left-side and the right-side limits.
3. Compare the left-side and the right side limits to conclude about the limit of the function at a.
4. Determine value of $f(a)$.
5. If $\lim_{x\to a} f(x) = f(a)$ then the function is continuous at $x = a$.

Ex. 5 Consider $f(x) = \begin{cases} x^2 - 1 & , x < 2 \\ -2x + 6, & x \geq 2 \end{cases}$



Determine $\lim_{x\to 2} f(x)$. Is the function continuous?