MCV 4U Name__________________________Date____________________

Velocity and Other Rates of Change

Average Rate of Change	**Instantaneous Rate of Change**
a. If the slope of a secant is considered the **average rate of change** of y with respect to the change in x then Slope of secant = $\frac{\Delta y}{\Delta x}$ = $\frac{f(x+h)-f(x)}{h}$ Or $\frac{f(x)-f(a)}{x-a}$	The slope of the tangent is then the **instantaneous rate** of change of y with respect to x Slope of tangent = $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ Or = $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$
b. Similarly, if average velocity is the rate of change of distance (s) with respect to time (t) then Average Velocity =	Instantaneous Velocity =
c. Average rate of change of temperature (T) with respect to time (t) =	Instantaneous rate of change of temperature with respect to time =
d. Average rate of change of volume (V) with respect to radius (r) =	Instantaneous rate of change of volume with respect to radius =

Now try…

1. The price $P(t)$ of one share in an oil company at time t is given by the formula $P(t) = -t^2 + 14t + 10$. The price is measured in dollars and the time in years.

a. Determine the average rate of change in price from $t = 1$ to $t = 5$ years.

b. Find the instantaneous price change at 5 years. ($ 8/year,$4/year)

2. Water is being pumped from a tank so that the volume V in litres remaining after t minutes is determined by $V(t) = 1000(10 - t^2)$ for $0 \le t \le \sqrt{10}$. Over the first 2 minutes what is the average rate of change of the volume remaining? Find the rate of change of volume with respect to time after 2 minutes.
(-2000L/min, -4000L/min)

3. In a contest to determine who can throw a ball straight up the greatest distance, a motion detector is used to measure the position of the ball. The height, in metres, of the ball above the ground is found by regression to be represented by the equation $s = 2 + 30t - 5t^2$, where t is measured in seconds.

a) Find the average velocity of the ball, beginning when $t = 3$ and lasting
i) 1 s **ii)** 0.5 s **iii)** 0.1 s **iv)** 0.01 s

b) Find the instantaneous velocity of the ball when $t = 3$.

c) Find the maximum height of the ball.

d) Find the velocity of the ball at $t = a$, where $a = 1,2,3,4,5,6$

e) Find the velocity of the ball when it returns to the height it was thrown from.
(-5, -2.5, -0.5, -0.05 m/s, 0 m/s, 30-10a, -30 m/s)

4. A spherical balloon is being inflated. $V = \dfrac{4\pi r^3}{3}$

a. Find the rate of change of volume with respect to radius when $r \in [1,3]$ cm.

b. Find the exact rate of change of volume with respect to radius when $r = 10$ cm. ($\approx 54.45 cm^3/cm, 400\pi\ cm^3/cm$)

5. A designer is experimenting with a cylindrical can with fixed height of 15 cm. Find the exact rate of change of volume with respect to radius when the radius is 4 cm. $V = \pi r^2 h$ ($120\pi\ cm^3/cm$)

6. A bicycle rider travelling along a straight road applies the brakes, and the rider's position in metres at any time t seconds is given by $s(t) = 3t - 0.75t^2$. What is the rider's initial velocity? How long does it take the rider to stop? (3m/s, 2s)

7. The period $T(x)$ of pendulum depends upon its length x. If the period is measured in seconds and the length in metres, $T(x) = 6.2\sqrt{0.1x}$, what is the rate of change with respect to length when the length is 250 cm? (0.062 m/s)

8. Find the rate of change of the area of an equilateral triangle with respect to change in its side length b. $A = \dfrac{bh}{2}$ Hint: Represent area in terms of b only.

($\dfrac{b\sqrt{3}}{2}$)